AD-A119 440 SALFORD UNIV (ENGLAND) DEPT OF AERONAUTICAL AND MECH--ETC F/G 17/7
DESIGN OF HIGH-PERFORMANCE TRACKING SYSTEMS.(U)
JUL 82 B PORTER F49620-81-C-0026
UNCLASSIFIED USAME/DC/120/81 AFWAL-TR-82-3032 NL

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AD A119440

AFWAL-TR-82-3032

DESIGN OF HIGH-PERFORMANCE TRACKING SYSTEMS

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JULY 82

TECHNICAL REPORT FOR PERIOD 15 NOVEMBER 1980 - 30 SEPTEMBER 1981

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AIR FORCE SYSTEMS COMMAND
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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. Report Number	2. Govt Accession No. AD-A119440	3. Recipient's Catalog Number
4. Title (and Subtitle) DESIGN OF HIGH-PERFORMANCE TRACKING SYSTEMS		5. Type of Report & Period Covered Scientific Report 15 November 1980 - 30 September 1981.
		6. Performing Org. Report Number USAME/DC/120/81
7. Author(s) Professor Brian Porter		8. Contract or Grant Number F49620-81-C-0026
9. Performing Organization Name and Address Department of Aeronautical & Mechanical Engineering University of Salford Salford M5 4WT, England		10. Program Element, Project, Task Area & Work Unit Numbers 62201F 2304/02
11. Controlling Office Name and Address AFWAL/FIGL Wright-Patterson AFB Ohio 45433, USA		12. Report Date 1 July 1982
		13. Number of Pages 167
14. Monitoring Agency Name and Address EOARD/LNV Department of the Air Force 223/231 Old Marylebone Road, London NW1 5TH		15.

16. & 17. Distribution Statement

Approved for public release; distribution unlimited.

18. Supplementary Notes

19. Key Words

Multivariable control systems; tracking systems; high-gain controllers; fast-sampling controllers; singular perturbation methods.

20. Abstract

Conceptually and computationally simple methods for the design of high-performance tracking systems are described. These methods are equally applicable to the design of both analogue and digital controllers, and are based upon the exploitation of a solitary system-theoretic result from the singular perturbation analysis of transfer function matrices. Illustrative examples involving the design of tracking systems incorporating both high-gain analogue controllers and fast-sampling digital controllers are presented.

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PREFACE

This research was sponsored by the Air Force Flight Dynamics Laboratory,(AFWAL/FIGL), Wright-Patterson AFB, United States Air Force, under Contract F49620-81-C-0026 and was performed in the Engineering Dynamics and Control Laboratory at the University of Salford. In this work, the significant conceptual contributions of Dr A Bradshaw and the computational assistance of Dr A Hemami are gratefully acknowledged. This report was expertly typed and collated by Mrs D Millward.

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ABBREVIATIONS AND SYMBOLS

A	plant matrix
B	input matrix
C	output matrix
F	measurement matrix
K_o, K_1	controller matrices
M	transducer matrix
T	sampling period
$CA^{k-1}B$	k-th Markov parameter
$G(\lambda)$	transfer function matrix
$\Gamma(\lambda)$	asymptotic transfer function matrix
e	error vector
f	sampling frequency
g	gain parameter
u	control input vector
v	command input vector
w	measurement vector
x	state vector
y	output vector
z	integral of error vector
$\mathcal{C}$	set of complex numbers
$\mathcal{R}$	set of real numbers
$\mathcal{R}^{pxq}$	set of real pxq matrices
$\mathcal{T}$	continuous-time set $[0,+\infty)$
$\mathcal{T}_T$	discrete-time set $\{0,T,2T,...\}$
$\mathcal{Z}_t$	set of transmission zeros
$\mathcal{Z}_s$	set of 'slow' modes
$\mathcal{Z}_f$	set of 'fast' modes

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

In spite of the extensive effort which has been expended during the past twenty years, most currently available techniques (Athans and Falb 1966, Rosenbrock 1974, MacFarlane 1980, Wonham 1974, Wolovich 1974, Davison and Ferguson 1981) for the design of tracking systems incorporating linear multivariable plants are both conceptually and computationally rather complicated. Furthermore, such techniques are almost exclusively concerned with the design of analogue controllers and are therefore inapplicable to the much more important practical task of designing digital controllers.

The design techniques described in this report are both conceptually and computationally simple. In addition, these techniques are equally applicable to the design of both analogue and digital controllers. This universality and simplicity derives from the fact that these techniques are based upon the systematic exploitation of a solitary system-theoretic result from the theory of singular perturbations (Porter and Shenton 1975). This system-theoretic result exhibits the distinctive asymptotic structure of the transfer function matrices of linear multivariable systems with 'slow' and 'fast' modes in a manner which is directly applicable to the design of tracking systems incorporating either high-gain analogue controllers or fast-sampling digital controllers.

1.2 REGULAR AND IRREGULAR PLANTS

In the context of the design of high-performance tracking systems, it is convenient to classify linear multivariable plants as follows:

(i) Regular plants: first Markov parameter of maximal rank, minimum phase;

(ii) Irregular plants: first Markov parameter of non-maximal rank, non-minimum phase.

The design of tracking systems incorporating regular plants is considered in Chapters 2 and 3, whilst the design of tracking systems incorporating irregular plants is considered in Chapters 4 and 5. In the case of regular plants, arbitrarily fast and non-interacting tracking behaviour is achievable by appropriate selection of gain parameters or sampling periods simply by implementing error-actuated analogue or digital proportional-plus-integral controllers. However, in the case of irregular plants, good tracking behaviour is achievable only by the simultaneous implementation of inner-loop compensators to provide appropriate extra measurements for control purposes.

This result is crucially important since it makes explicit the intuitively obvious fact that controller and transducer designs are inseparable, and confirms the fact that the irrelevance of much of 'modern' control theory to practical engineering design derives from its failure to consider integrated controller/transducer systems. The importance of the difference between regular and irregular plants obviously requires that the set of transmission zeros of linear multivariable plants (Porter and D'Azzo 1977) be readily computable and very powerful software for this purpose has accordingly been developed (Porter 1979).

CHAPTER 2

DESIGN OF TRACKING SYSTEMS INCORPORATING HIGH-GAIN ERROR-ACTUATED CONTROLLERS

2.1 Introduction

In this chapter, singular perturbation methods are used to exhibit the asymptotic structure of the transfer function matrices of continuous-time tracking systems incorporating linear multivariable plants which are amenable to high-gain error-actuated analogue control (Porter and Bradshaw 1979a). Such tracking systems consist of a linear multivariable plant governed on the continuous-time set $T = [0,+\infty)$ by state and output equations of the respective forms (Porter and Bradshaw 1979b)

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} , & A_{12} \\ A_{21} , & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u(t) \qquad (2.1)$$

and

$$y(t) = [C_1 , C_2] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} , \qquad (2.2)$$

together with a high-gain error-actuated analogue controller governed on T by a control-law equation of the form

$$u(t) = g\{K_0 e(t) + K_1 z(t)\} \qquad (2.3)$$

which is required to generate the control input vector $u(t)$ so as to cause the output vector $y(t)$ to track any constant command input vector $v(t)$ on T in the sense that the error vector $e(t) = v(t) - y(t)$ assumes the steady-state value

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty}\{v(t)-y(t)\} = 0 \qquad (2.4)$$

for arbitrary initial conditions. In equations (2.1),(2.2),(2.3), and (2.4), $x_1(t) \in R^{n-\ell}$, $x_2(t) \in R^{\ell}$, $u(t) \in R^{\ell}$, $y(t) \in R^{\ell}$, $A_{11} \in R^{(n-\ell)x(n-\ell)}$, $A_{12} \in R^{(n-\ell)x\ell}$, $A_{21} \in R^{\ell x(n-\ell)}$, $A_{22} \in R^{\ell x\ell}$, $B_2 \in R^{\ell x\ell}$, $C_1 \in R^{\ell x(n-\ell)}$, $C_2 \in R^{\ell x\ell}$, rank $C_2B_2 = \ell$, $e(t) \in R^{\ell}$, $v(t) \in R^{\ell}$, $z(t) = z(o) + \int_0^t e dt \in R^{\ell}$, $K_o \in R^{\ell x\ell}$, $K_1 \in R^{\ell x\ell}$, and $g \in R^{+}$.

It is evident from equations (2.1),(2.2),(2.3) that such continuous-time tracking systems are governed on T by state and output equations of the respective forms

$$\begin{bmatrix} \dot{z}(t) \\ \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & , & -C_1 & , & -C_2 \\ 0 & , & A_{11} & , & A_{12} \\ gB_2K_1 & , & A_{21}-gB_2K_oC_1 & , & A_{22}-gB_2K_oC_2 \end{bmatrix} \begin{bmatrix} z(t) \\ x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} I_\ell \\ 0 \\ gB_2K_o \end{bmatrix} v(t) \tag{2.5}$$

and

$$y(t) = [0 \ , \ C_1 \ , \ C_2] \begin{bmatrix} z(t) \\ x_1(t) \\ x_2(t) \end{bmatrix} \quad . \tag{2.6}$$

The transfer function matrix relating the plant output vector to the command input vector of the closed-loop continuous-time tracking system governed by equations (2.5) and (2.6) is clearly

$$G(\lambda)=[0,C_1,C_2]\begin{bmatrix}\lambda I_\ell\ , & C_1\ , & C_2\\ 0\ , & \lambda I_{n-\ell}-A_{11}\ , & -A_{12}\\ -gB_2K_1, & -A_{21}+gB_2K_oC_1, & \lambda I_\ell-A_{22}+gB_2K_oC_2\end{bmatrix}^{-1}\begin{bmatrix}I_\ell\\ 0\\ gB_2K_o\end{bmatrix}$$

(2.7)

and the high-gain tracking characteristics of this sytem can accordingly be elucidated by invoking the results of Porter and Shenton (1975) from the singular perturbation analysis of transfer function matrices.

These results yield the asymptotic form of $G(\lambda)$ as the gain parameter $g \to \infty$ and thus greatly facilitate the determination of controller matrices K_o and K_1 such that the continuous-time tracking behaviour of the closed-loop system becomes increasingly 'tight' and non-interacting as g is increased. The frequency-response and step-response characteristics of a continuous-time tracking system incorporating an open-loop unstable chemical reactor (MacFarlane and Kouvaritakis 1977) are presented in order to illustrate these general results.

2.2 ANALYSIS

It is evident from equation (2.7) that, by regarding $\varepsilon = 1/g$ as the perturbation parameter, the asymptotic form of the transfer function matrix $G(\lambda)$ of the continuous-time tracking system as $g \to \infty$ can be determined by invoking the results of Porter and Shenton (1975) from the singular perturbation analysis of transfer function matrices. Indeed, these results indicate that as $g \to \infty$ the transfer function matrix $G(\lambda)$ assumes the asymptotic form

$$\Gamma(\lambda) = \tilde{\Gamma}(\lambda) + \hat{\Gamma}(\lambda) \tag{2.8}$$

where

$$\tilde{\Gamma}(\lambda) = C_o(\lambda I_n - A_o)^{-1}B_o \quad , \tag{2.9}$$

$$\hat{\Gamma}(\lambda) = C_2(\lambda I_\ell - gA_4)^{-1}gB_2K_o \quad , \tag{2.10}$$

$$A_o = \begin{bmatrix} -K_o^{-1}K_1 & , & 0 \\ A_{12}C_2^{-1}K_o^{-1}K_1 & , & A_{11}-A_{12}C_2^{-1}C_1 \end{bmatrix} \quad , \tag{2.11}$$

$$B_o = \begin{bmatrix} 0 \\ A_{12}C_2^{-1} \end{bmatrix} \tag{2.12}$$

$$C_o = [K_o^{-1}K_1 \ , \ 0] \quad , \tag{2.13}$$

and

$$A_4 = -B_2K_oC_2 \quad . \tag{2.14}$$

It follows from equations (2.8), (2.9), and (2.11) that the 'slow' modes Z_s of the tracking system correspond as $g \to \infty$ to the poles $Z_1 \cup Z_2$ of $\tilde{\Gamma}(\lambda)$ where

$$Z_1 = \{\lambda \in C : |\lambda K_o + K_1| = 0\} \tag{2.15}$$

and

$$Z_2 = \{\lambda \in C : |\lambda I_{n-\ell} - A_{11} + A_{12}C_2^{-1}C_1| = 0\} \quad , \tag{2.16}$$

and from equations (2.8), (2.10), and (2.14) that the 'fast' modes Z_f of the tracking system correspond as $g \to \infty$ to the poles Z_3 of $\hat{\Gamma}(\lambda)$ where

$$Z_3 = \{\lambda \in C : |\lambda I_\ell + gC_2B_2K_o| = 0\} \quad . \tag{2.17}$$

Furthermore, it follows from equations (2.9),(2.11),(2.12), and (2.13) that the 'slow' transfer function matrix

$$\tilde{\Gamma}(\lambda) = O \tag{2.18}$$

and from equation (2.10) and (2.14) that the 'fast' transfer function matrix

$$\hat{\Gamma}(\lambda) = (\lambda I_\ell + gC_2B_2K_o)^{-1} gC_2B_2K_o \quad . \tag{2.19}$$

Hence, in view of equations (2.18) and (2.19), it is evident from equation (2.8) that as $g \to \infty$ the transfer function matrix $G(\lambda)$ of the continuous-time tracking system assumes the asymptotic form

$$\Gamma(\lambda) = (\lambda I_\ell + gC_2B_2K_o)^{-1} gC_2B_2K_o \tag{2.20}$$

in consonance with the fact that only the 'fast' modes corresponding to the poles Z_3 remain both controllable and observable as $g \to \infty$. Indeed, the 'slow' transfer function matrix $\tilde{\Gamma}(\lambda)$ vanishes precisely because the 'slow' modes corresponding to the poles Z_1 become asymptotically uncontrollable as $g \to \infty$ in view of the block structure of the matrices A_o and B_o in equations (2.11) and (2.12) whilst the 'slow' modes corresponding to the poles Z_2 become asymptotically unobservable as $g \to \infty$ in view of the block structure of the matrices A_o and C_o in equations (2.11) and (2.13)

3. SYNTHESIS

It is evident from equations (2.5) and (2.6) that tracking will occur in the sense of equation (4) provided only that

$$Z_s \cup Z_f \subset C^- \tag{2.21}$$

where C^- is the open left half-plane. In view of equations (2.15),(2.16),and(2.17), the 'slow' and 'fast' modes will satisfy the tracking requirement (2.21) for sufficiently large gains if the controller matrices K_o and K_1 are chosen such that both $Z_1 \subset C^-$ and $Z_3 \subset C^-$ in the case of minimum-phase plants for which (Porter and D'Azzo 1977) the set of transmission zeros

$$Z_t = \{\lambda \in C : \lambda I_{n-\ell} - A_{11} + A_{12}C_2^{-1}C_1 | = 0\} \subset C^- \tag{2.22}$$

since it is then immediately obvious from equation (2.16) that $Z_2 \subset C^-$. Moreover, in such cases, tracking will become increasingly 'tight' as $g \to \infty$ in view of equation (2.20). Furthermore, if K_o is chosen such that

$$C_2B_2K_o = \mathrm{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_\ell\} \tag{2.23}$$

where $\sigma_j \in R^+$ $(j=1,2,\ldots,\ell)$, it follows from equation (2.20) that the transfer function matrix $G(\lambda)$ of the continuous-time tracking system will assume the diagonal asymptotic form

$$\Gamma(\lambda) = \mathrm{diag}\left\{\frac{g\sigma_1}{\lambda+g\sigma_1}, \frac{g\sigma_2}{\lambda+g\sigma_2}, \ldots, \frac{g\sigma_\ell}{\lambda+g\sigma_\ell}\right\} \tag{2.24}$$

and therefore that increasingly non-interacting tracking behaviour will occur as $g \to \infty$.

2.4 Illustrative Example

These general results can be conveniently illustrated by designing a high-gain error-actuated analogue controller for an open-loop unstable chemical reactor governed on T by the respective state and output equations (MacFarlane and Kouvaritakis 1977)

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix} = \begin{bmatrix} 1.38 & , -0.2077 & , 6.715 & , -5.676 \\ -0.5814 & , -4.29 & , 0 & , 0.675 \\ 1.067 & , 4.273 & , -6.654 & , 5.893 \\ 0.048 & , 4.273 & , 1.343 & , -2.104 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & , 0 \\ 5.679 & , 0 \\ 1.136 & , -3.146 \\ 1.136 & , 0 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \qquad (2.25)$$

and

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 , & 0, & 1 , & -1 \\ 0 , & 1 , & 0 , & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} \qquad (2.26)$$

from which it can be readily verified that $Z_t = \{-1.192, -5.039\} \subset C^-$ and that the first Markov parameter

$$C_2B_2 = \begin{bmatrix} 0 & -3.146 \\ 5.679, & 0 \end{bmatrix} \qquad (2.27)$$

is of full rank. In case $\{\sigma_1,\sigma_2\} = \{1,1\}$ and $K_1 = 2K_0$, it follows from equations (2.3), (2.23), and (2.27) that the corresponding high-gain error-actuated analogue controller is governed on by the control-law equation

$$\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = g\left\{\begin{bmatrix} 0 & ,0.1761 \\ -0.3179, & 0 \end{bmatrix}\begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} + \begin{bmatrix} 0 & ,0.3522 \\ -0.6358, & 0 \end{bmatrix}\begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix}\right\}$$

(2.28)

and it is evident from equations (2.17), (2.18), and (2.19) that $Z_1 = \{-2,-2\}$, $Z_2 = \{-1.192,-5.039\}$, and $Z_3 = \{-g,-g\}$. It is also evident from equation (2.24) that the asymptotic transfer function matrix assumes the diagonal form

$$\Gamma(\lambda) = \begin{bmatrix} \frac{g}{\lambda+g} & , 0 \\ 0 & , \frac{g}{\lambda+g} \end{bmatrix} \qquad (2.29)$$

and therefore that the closed-loop continuous-time tracking system incorporating the chemical reactor will exhibit increasingly 'tight' and non-interacting tracking behaviour as $g \to \infty$ when the control input vector $[u_1(t)\ ,\ u_2(t)]^T$ is generated by the high-gain analogue controller governed on T by equation (2.28).

The actual frequency-response loci $G(i\omega)$ for $\omega \in (-\infty,+\infty)$ are shown in Figs 2.1, 2.2, and 2.3 when $g = 25$, 50, and 100, respectively and it is clear that the actual frequency-response loci approach the asymptotic frequency-response loci $\Gamma(i\omega)$ as the gain parameter g is increased. The corresponding step-response characteristics are shown in Fig 2.4 and Fig 2.5 when $[v_1(t) \ , \ v_2(t)]^T = [1 \ , \ 0]^T$ and $[v_1(t) \ , \ v_2(t)]^T = [0 \ , \ 1]^T$, respectively, and it is evident that increasingly 'tight' and non-interacting tracking occurs as the gain parameter g is increased.

2.5 Conclusion

Singular perturbation methods have been used to exhibit the asymptotic structure of the transfer function matrices of continuous-time tracking systems incorporating linear multivariable plants which are amenable to high-gain error-actuated analogue control. It has been shown that these results greatly facilitate the determination of controller matrices which ensure that the closed-loop behaviour of such continuous-time tracking systems becomes increasingly 'tight' and non-interacting as the gain parameter g is increased. These general results have been illustrated by the presentation of the frequency-response and step-response characteristics of a continuous-time tracking system incorporating an open-loop unstable chemical reactor.

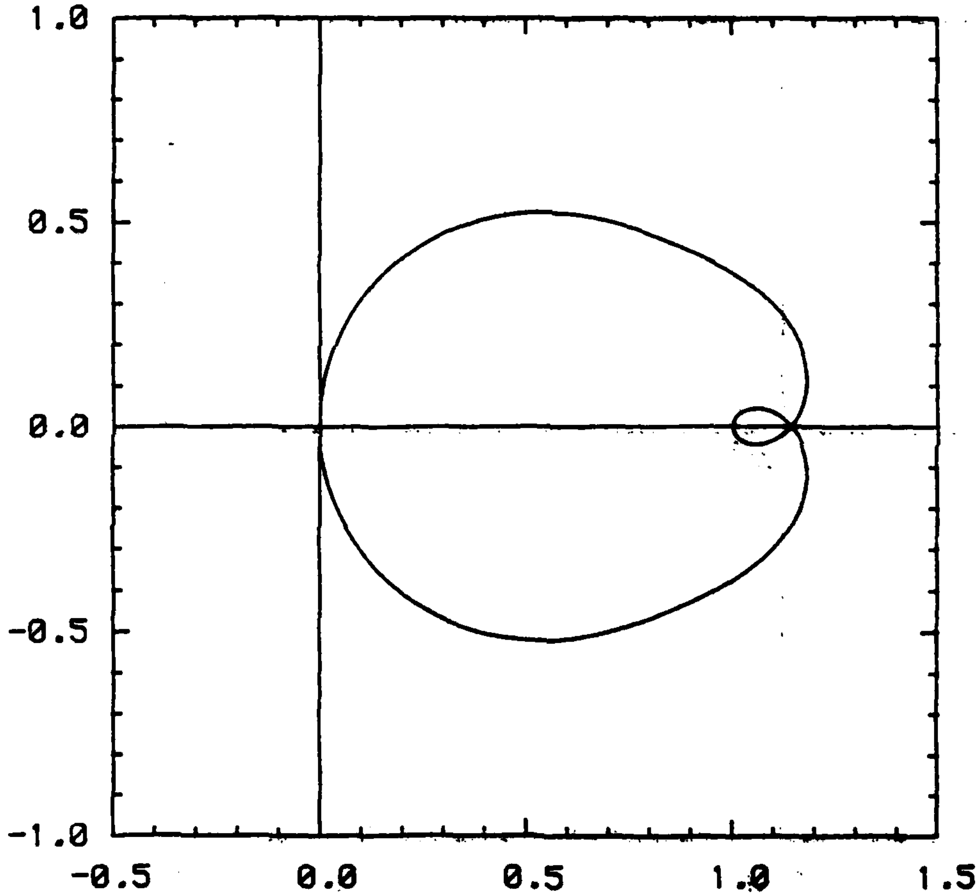


Fig 2.1(a): $G_{11}(i\omega)$; $g = 25$

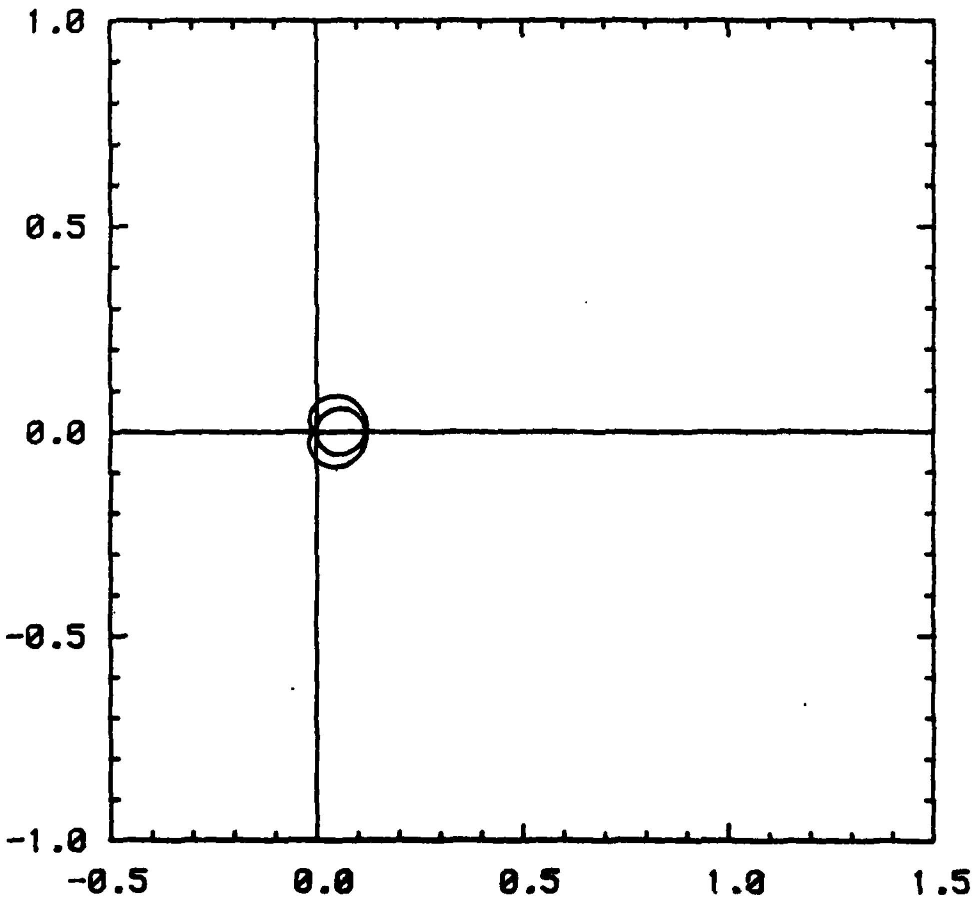


Fig 2.1(b): $G_{12}(i\omega)$; $g = 25$

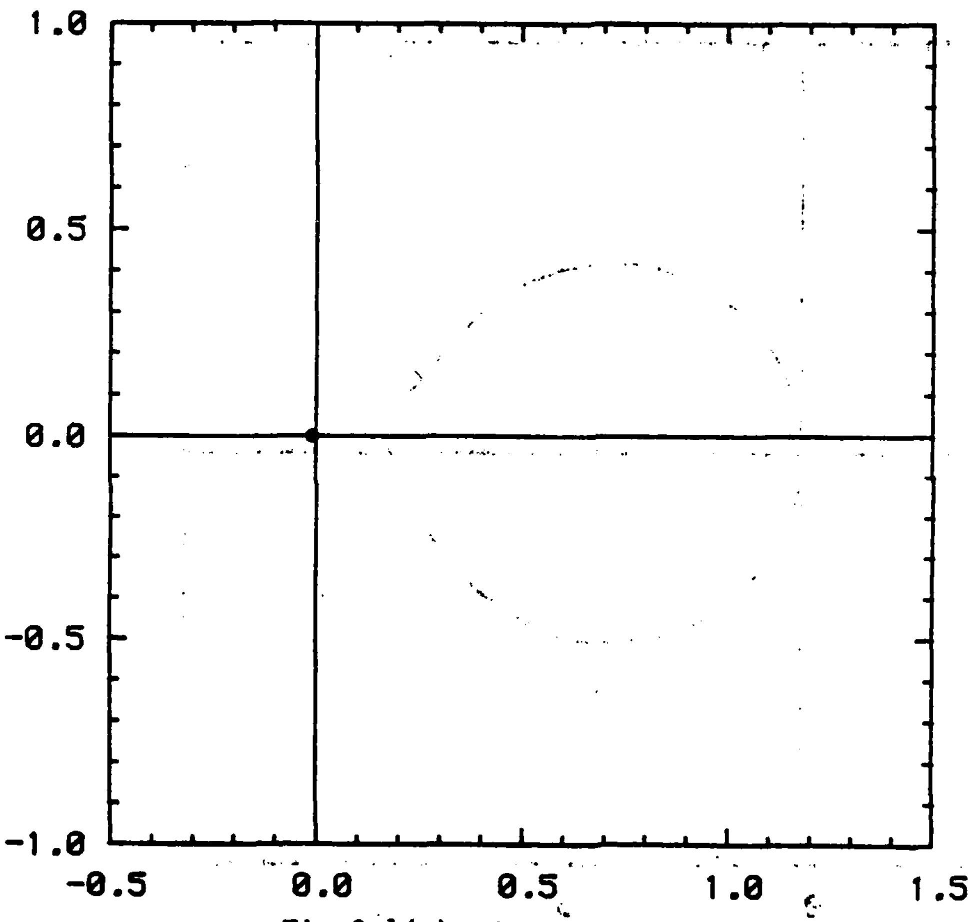


Fig 2.1(c): $G_{21}(i\omega)$; g = 25

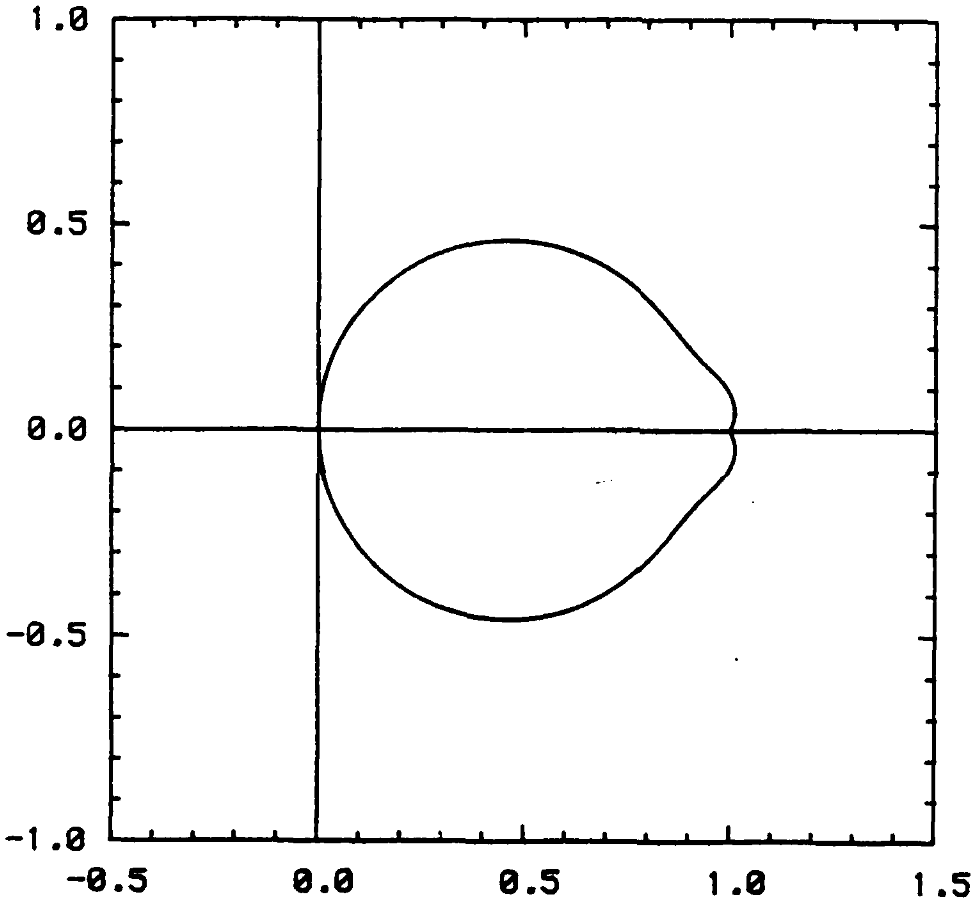


Fig 2.1(d): $G_{22}(i\omega)$; g = 25

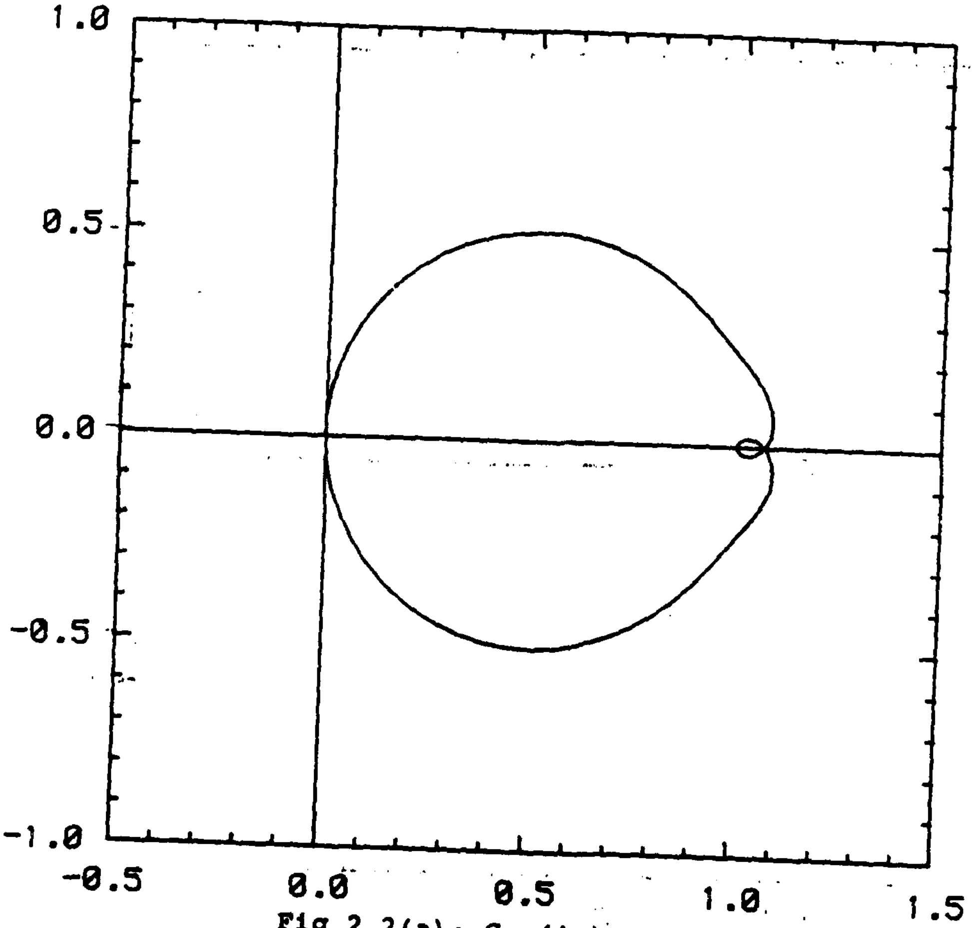


Fig 2.2(a): $G_{11}(i\omega)$; g = 50

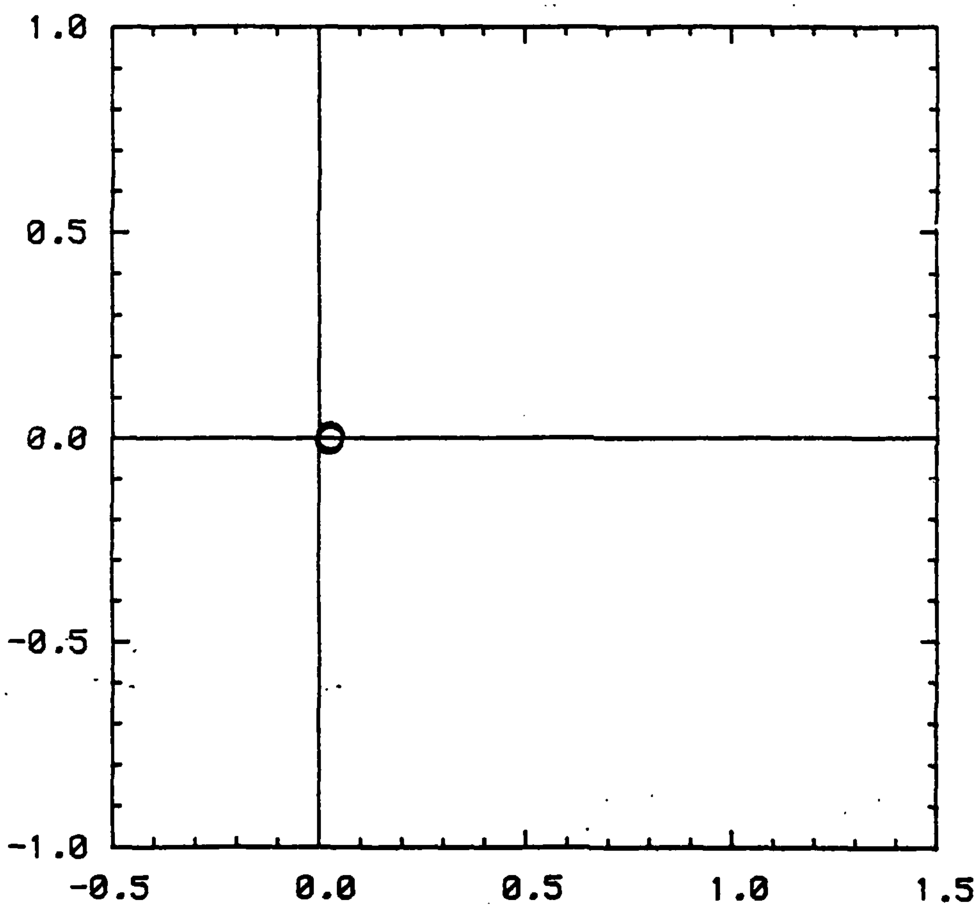


Fig 2.2(b): $G_{12}(i\omega)$; $g = 50$

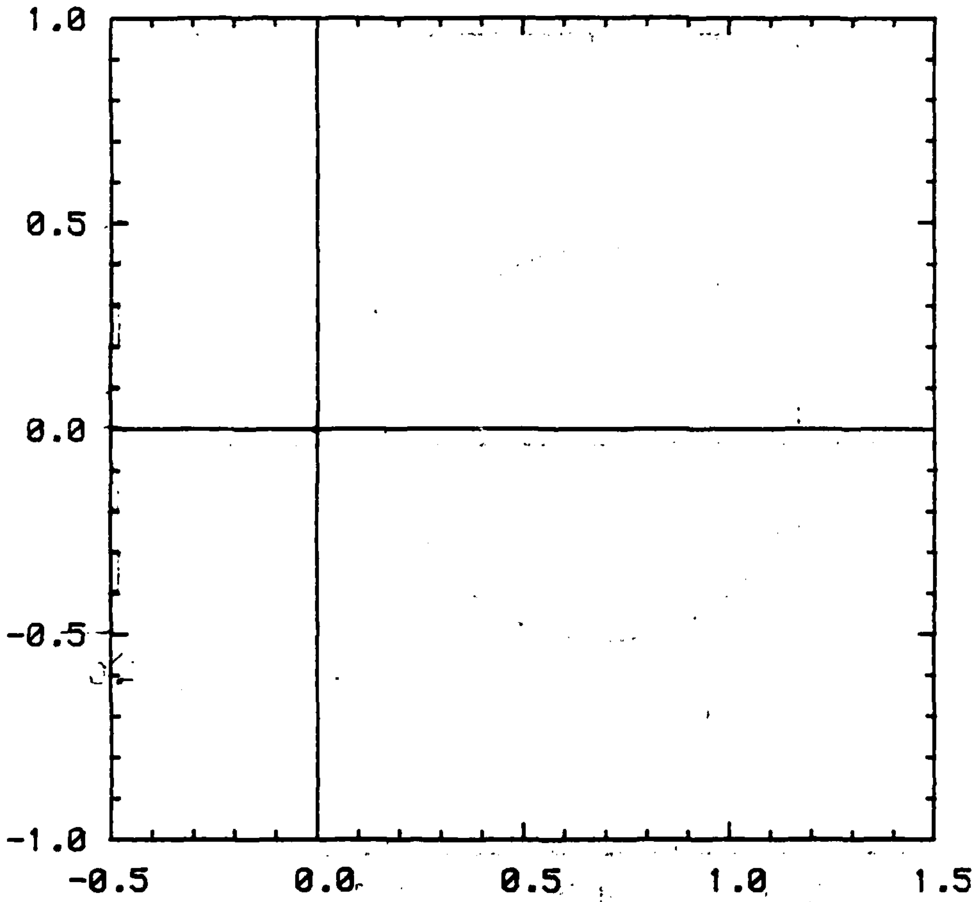


Fig 2.2(c): $G_{21}(i\omega)$; g = 50

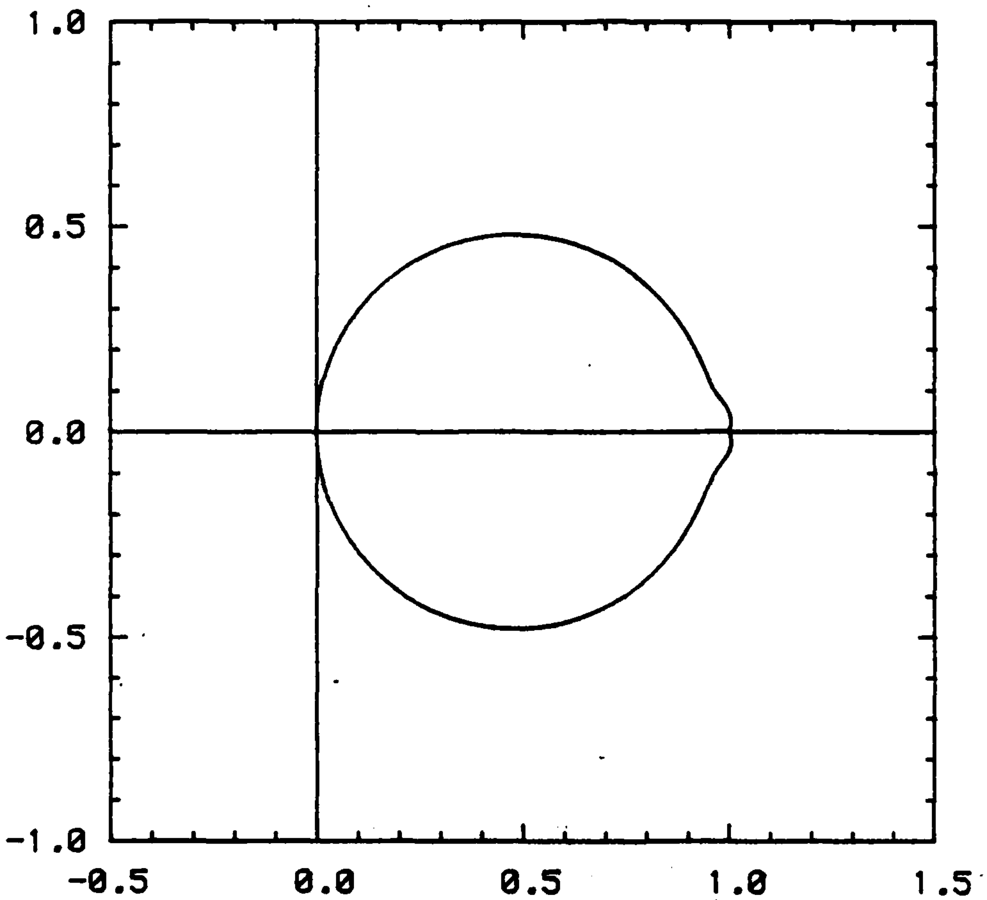


Fig 2.2(d): $G_{22}(i\omega)$; g = 50

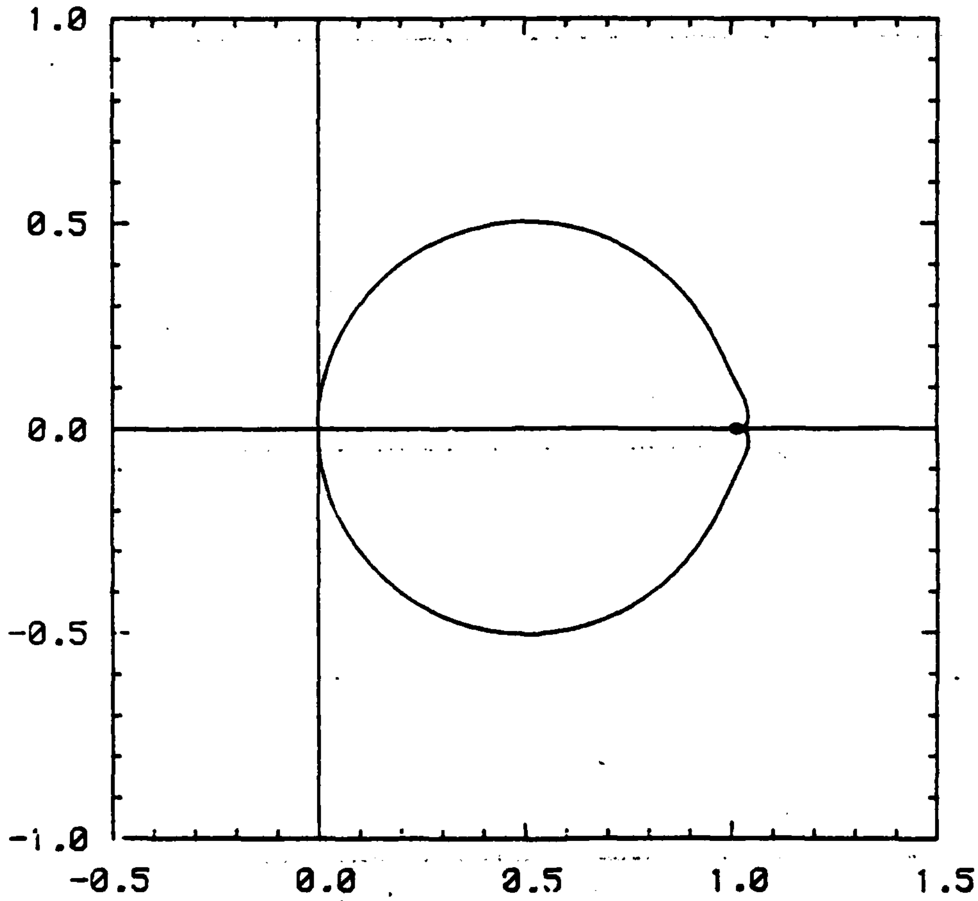


Fig 2.3(a): $G_{11}(i\omega)$; $g = 100$

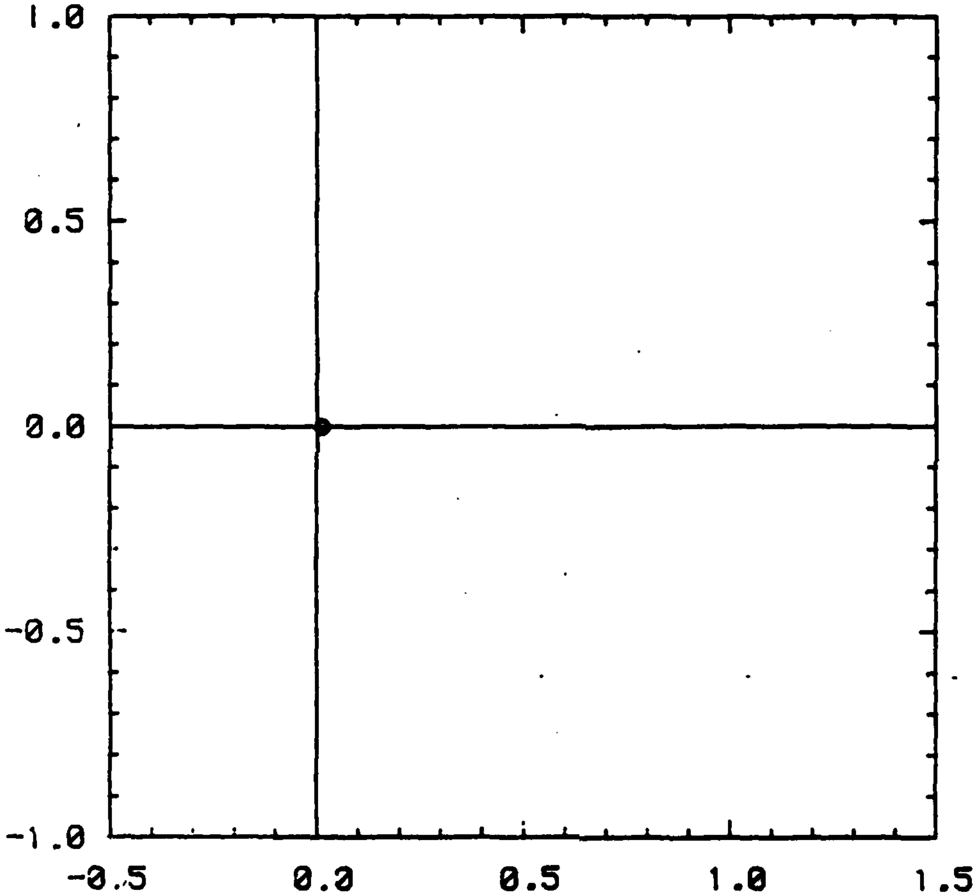


Fig 2.3(b): $G_{12}(i\omega)$; $g = 100$

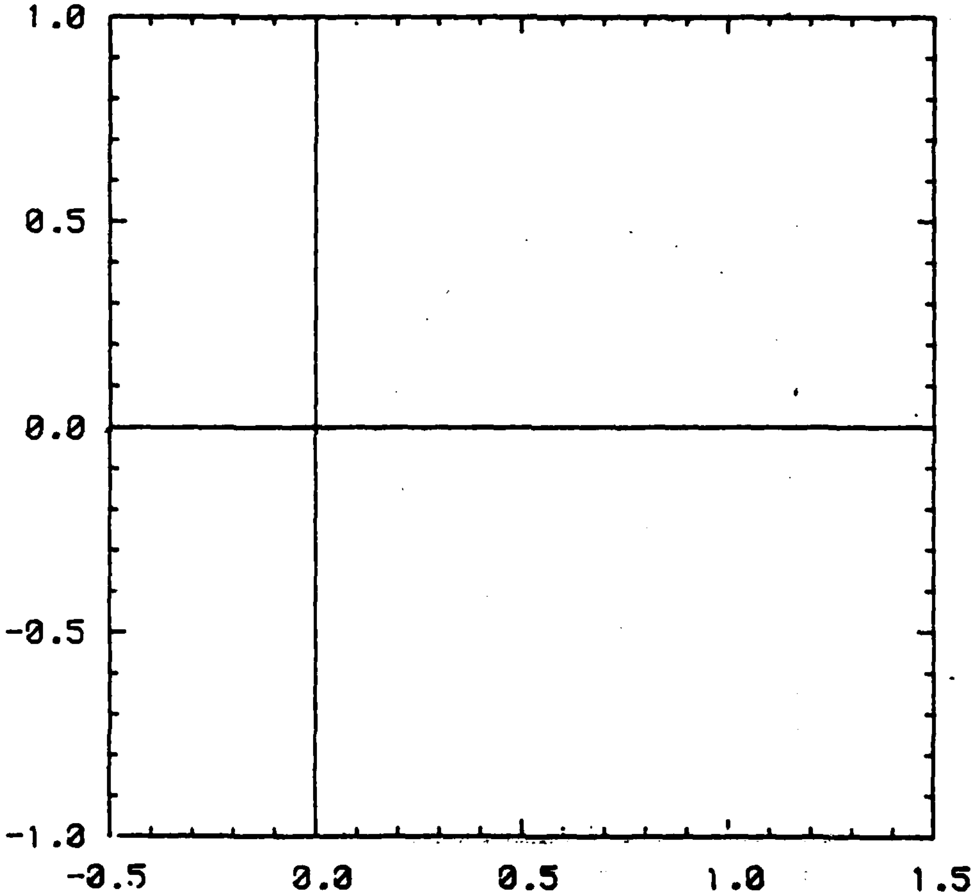


Fig 2.3(c): $G_{21}(i\omega)$; $g = 100$

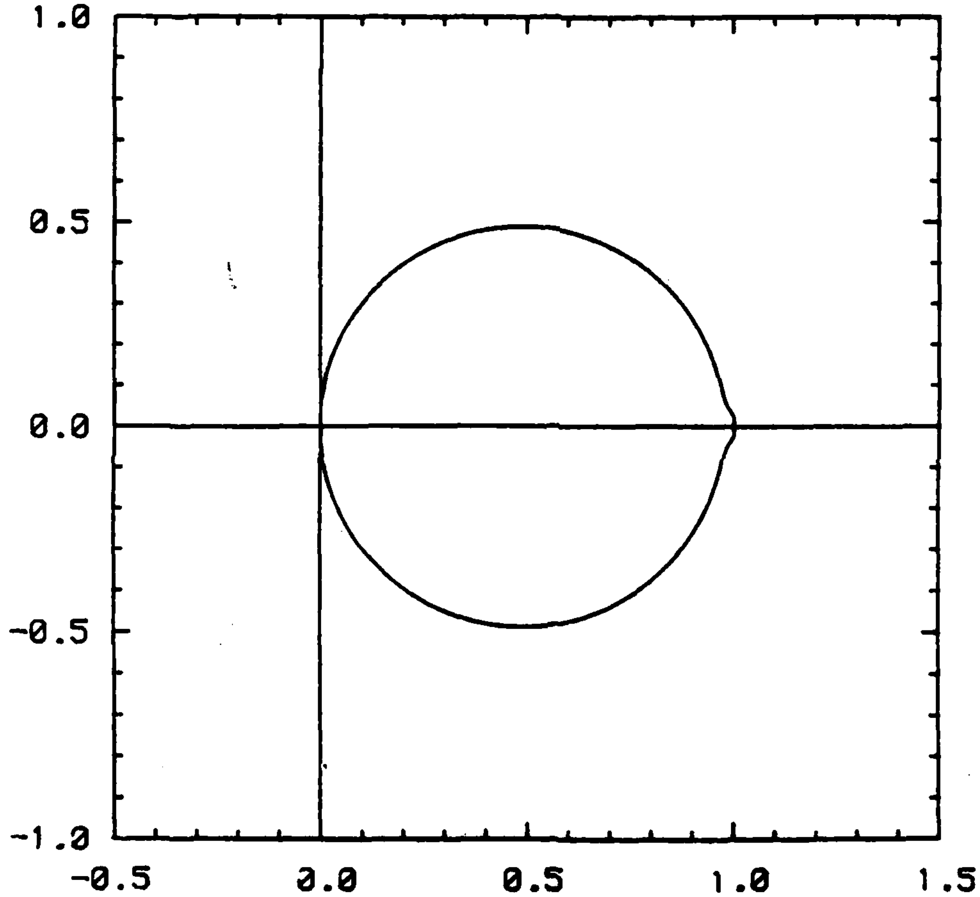


Fig 2.3(d): $G_{22}(i\omega)$; $g = 100$

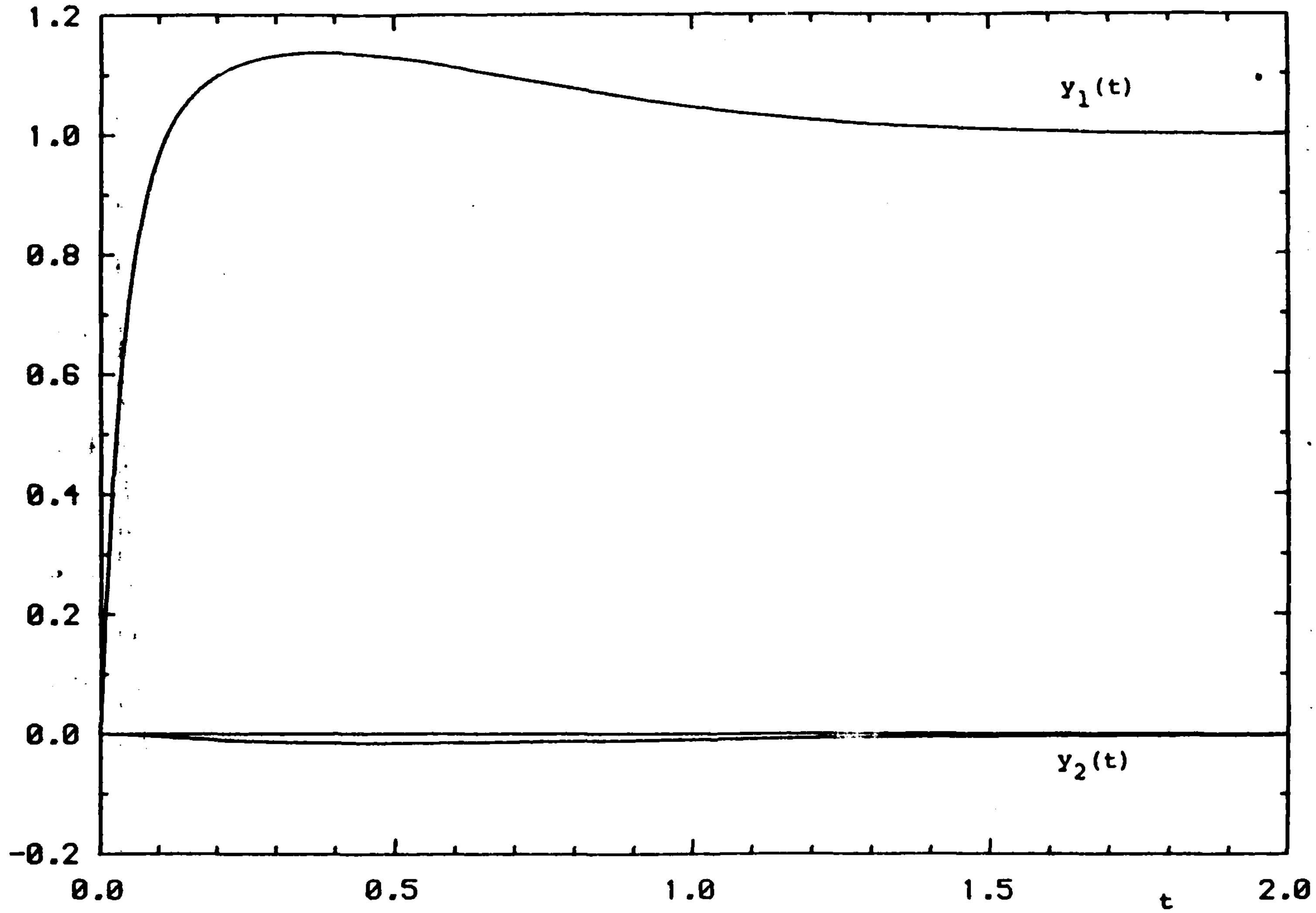


Fig 2.4(a): g = 25

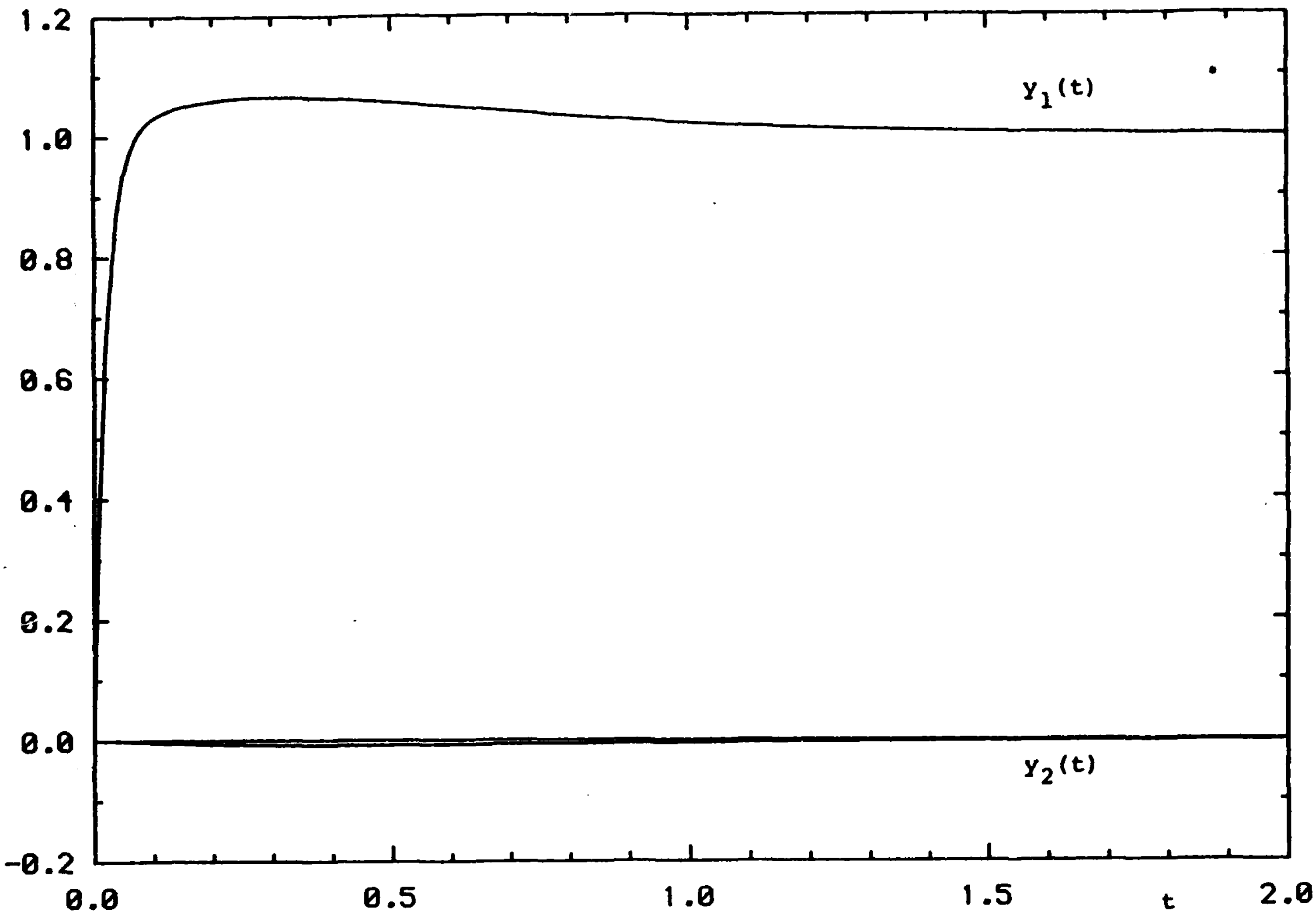


Fig 2.4(b): g = 50

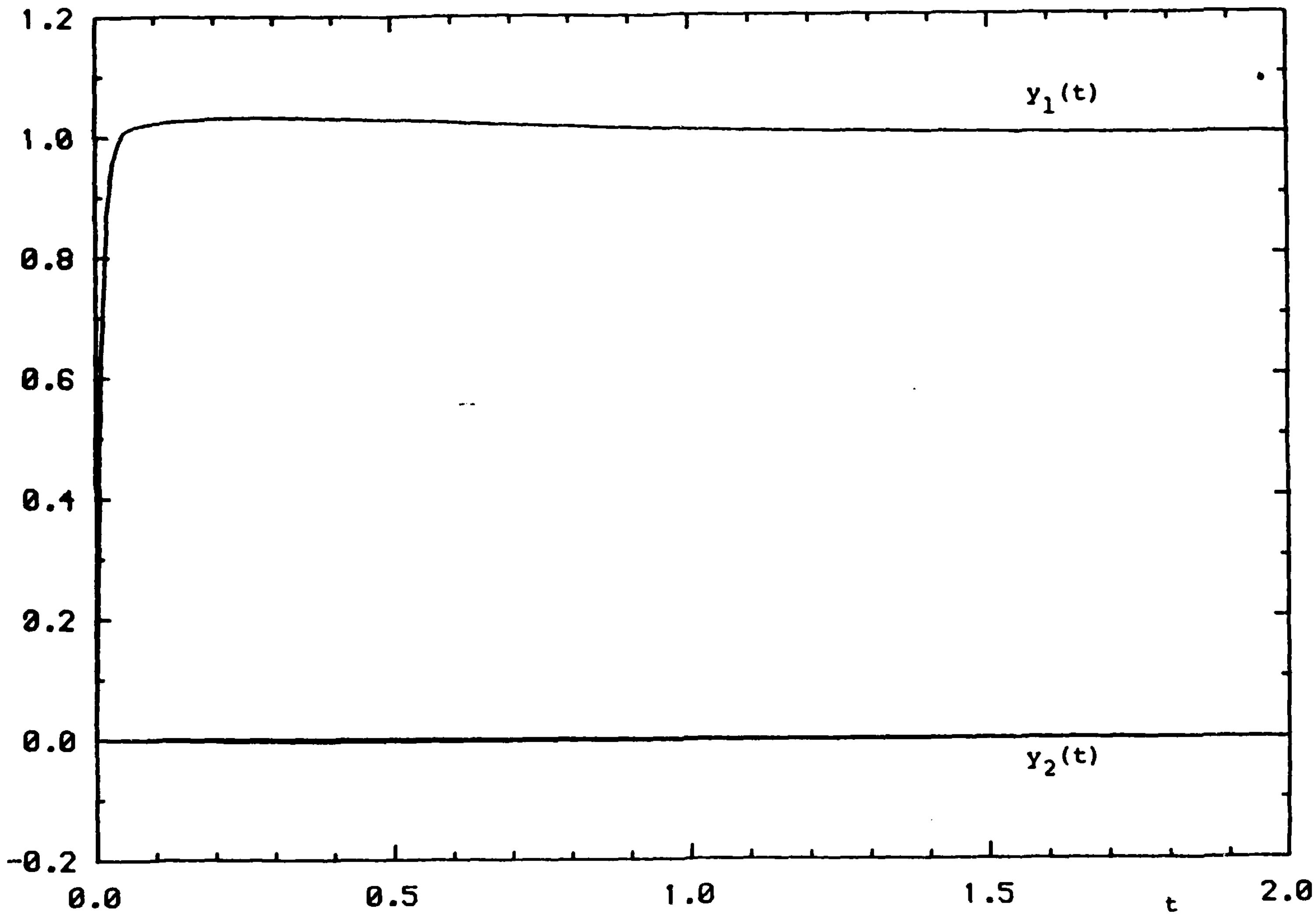


Fig 2.4(c): g = 100

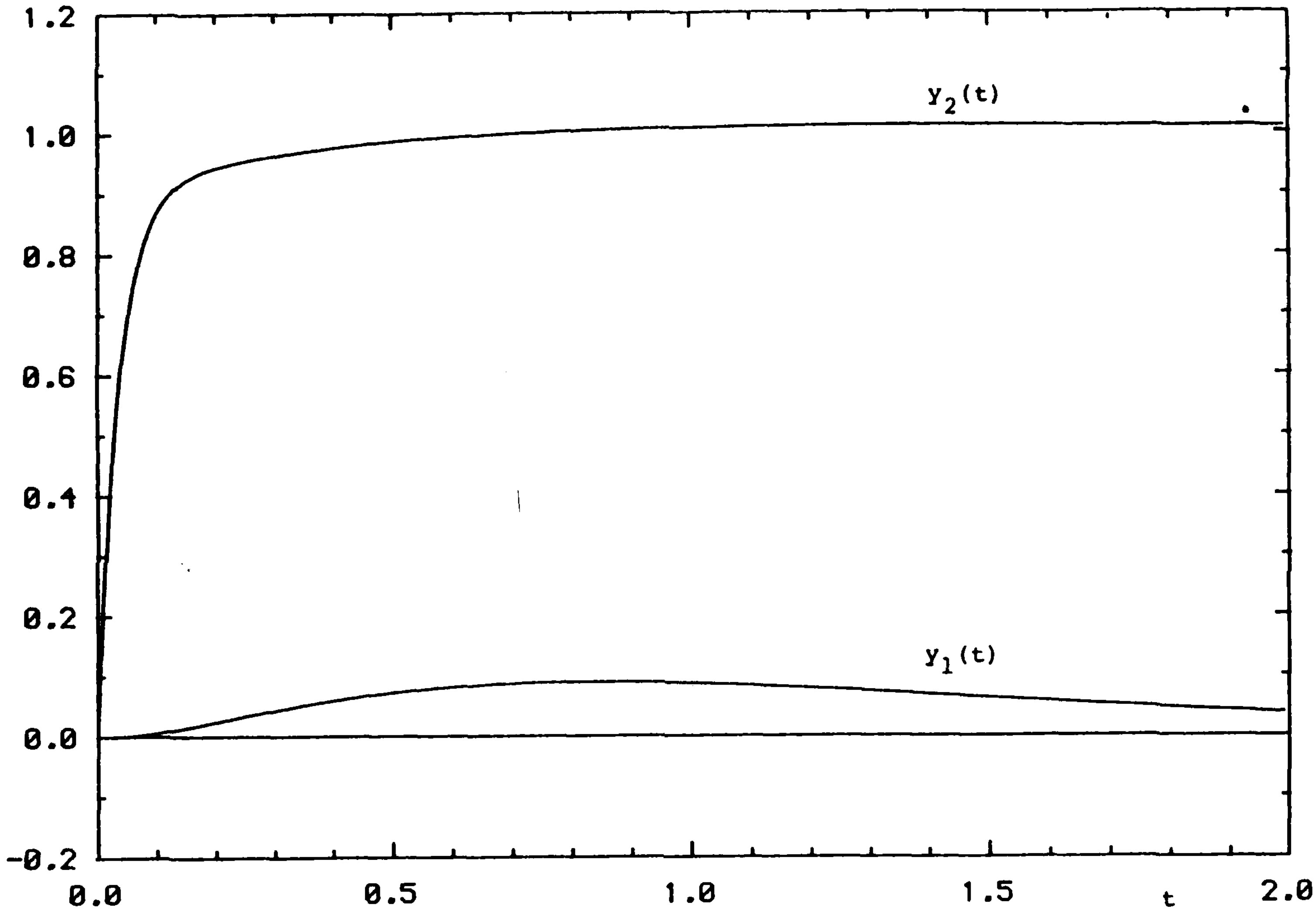


Fig 2.5(a): g = 25

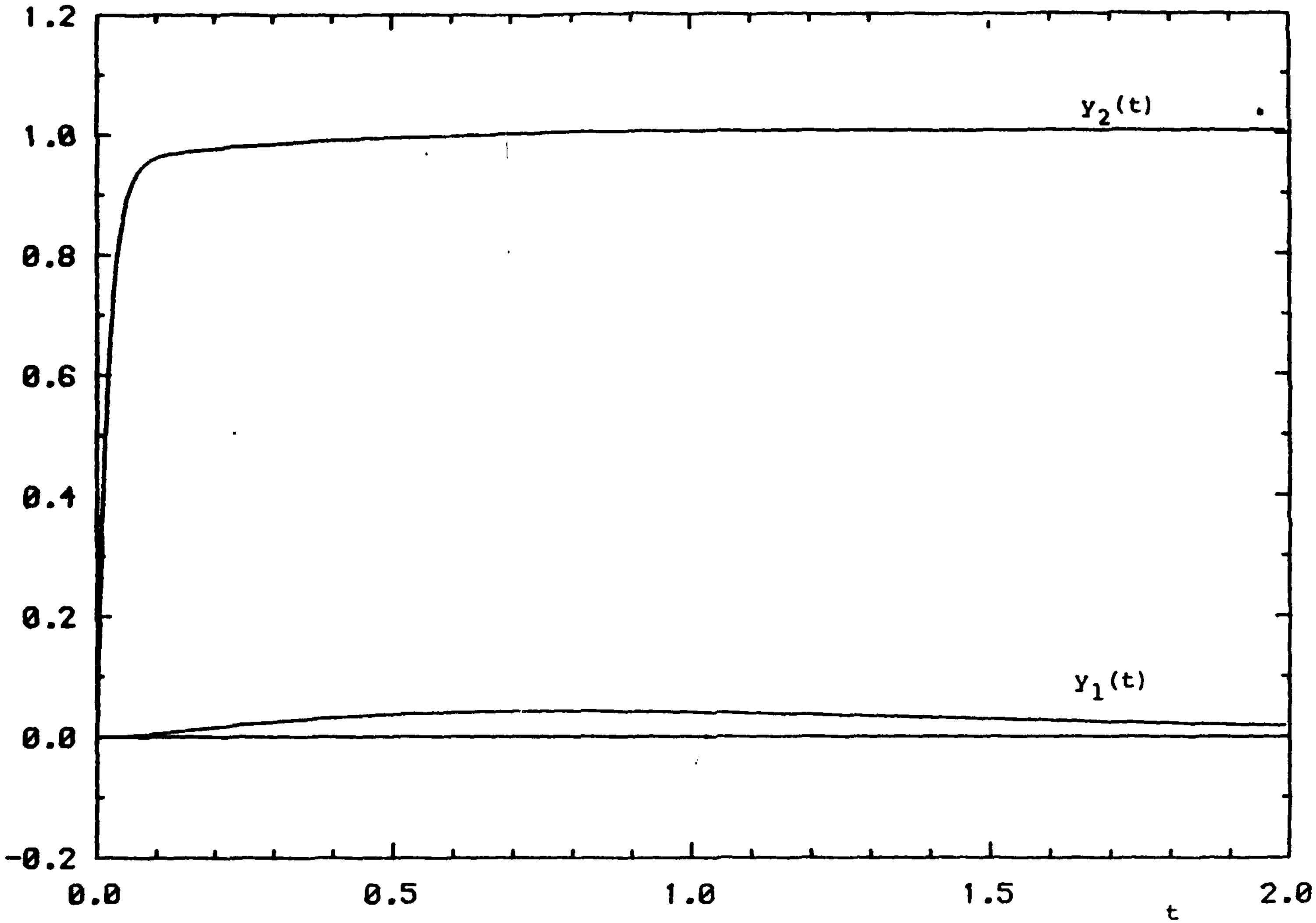
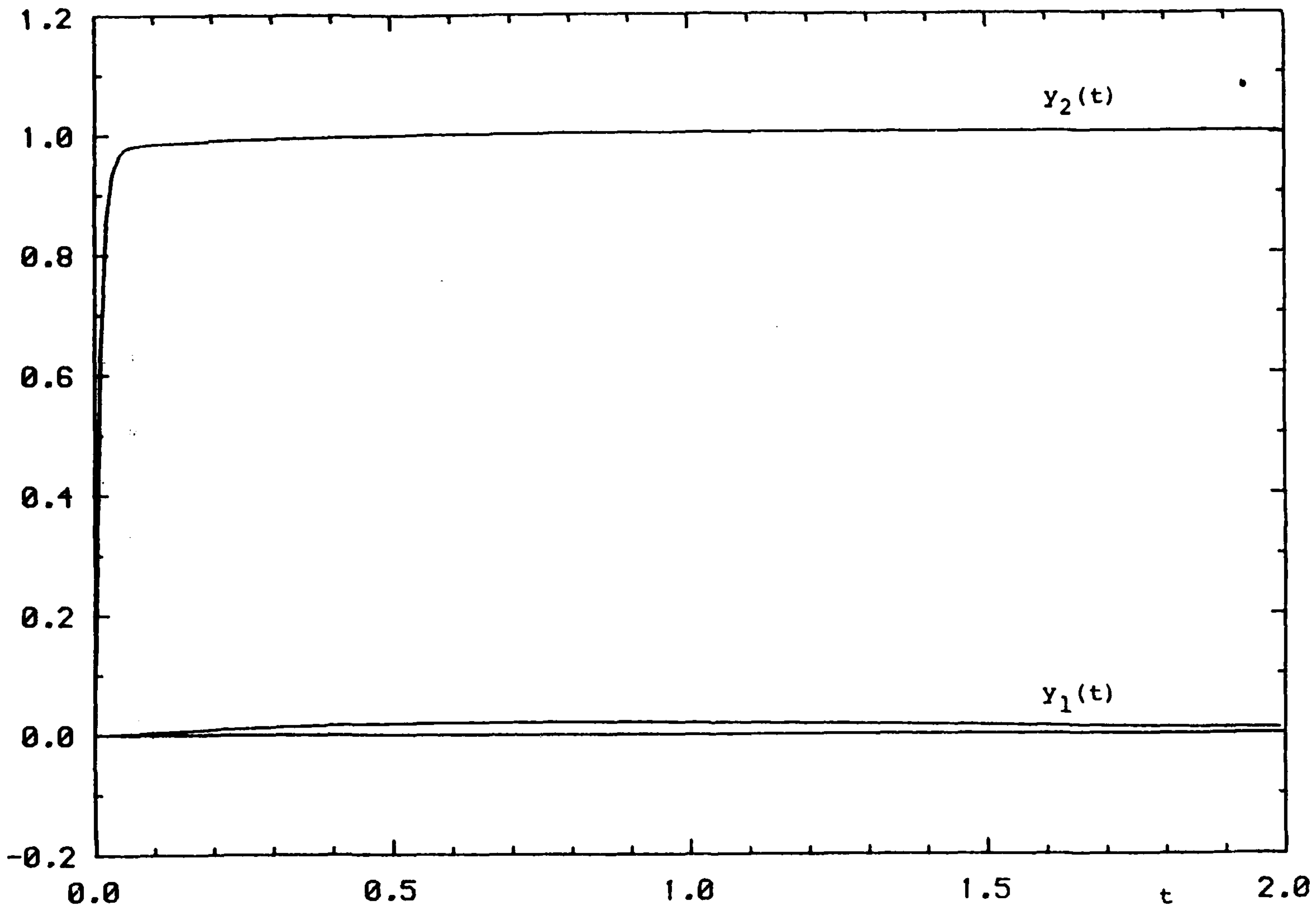


Fig 2.5(b): g = 50



Fug 2.5(c): g = 100

CHAPTER 3

DESIGN OF TRACKING SYSTEMS INCORPORATING FAST-SAMPLING ERROR-ACTUATED CONTROLLERS

3.1 INTRODUCTION

In this chapter, singular perturbation methods are used to exhibit the asymptotic structure of the transfer function matrices of discrete-time tracking systems incorporating linear multivariable plants which are amenable to fast-sampling error-actuated control (Bradshaw and Porter 1980a). Such tracking systems consist of a linear multivariable plant governed on the continuous-time set $T = [0,+\infty)$ by state and output equations of the respective forms (Bradshaw and Porter 1980b)

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} , A_{12} \\ A_{21} , A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u(t) \tag{3.1}$$

and

$$y(t) = [C_1 , C_2] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} , \tag{3.2}$$

together with a fast-sampling error-actuated digital controller governed on the discrete-time set $T_T = \{0,T,2T,...\}$ by a control-law equation of the form

$$u(kT) = f\{K_o e(kT) + K_1 z(kT)\} \tag{3.3}$$

which is required to generate the piecewise-constant control input vector $u(t) = u(kT)$, $t \in [kT,(k+1)T)$, $kT \in T_T$, so as to cause the output vector $y(t)$ to track any constant command input vector $v(t)$ on T_T in the sense that the error vector $e(t) = v(t) - y(t)$ assumes the steady-state value

$$\lim_{k\to\infty} e(kT) = \lim_{k\to\infty}\{v(kT)-y(kT)\} = 0 \tag{3.4}$$

for arbitrary initial conditions where $f = 1/T$ and T is the sampling period. In equations (3.1),(3.2),(3.3), and (3.4), $x_1(t) \in R^{n-\ell}$, $x_2(t) \in R^{\ell}$, $u(t) \in R^{\ell}$, $y(t) \in R^{\ell}$, $A_{11} \in R^{(n-\ell)x(n-\ell)}$, $A_{12} \in R^{(n-\ell)x\ell}$, $A_{21} \in R^{\ell x(n-\ell)}$, $A_{22} \in R^{\ell x\ell}$, $B_2 \in R^{\ell x\ell}$, $C_1 \in R^{\ell x(n-\ell)}$, $C_2 \in R^{\ell x\ell}$, rank $C_2B_2 = \ell$, $e(t) \in R^{\ell}$, $v(t) \in R^{\ell}$, $z(kT) = z(o) + T\sum_{j=0}^{j=k-1} e(jT) \in R^{\ell}$, $K_o \in R^{\ell x\ell}$, $K_1 \in R^{\ell x\ell}$, and $f \in R^{+}$.

It is evident from equations (3.1),(3.2), and (3.3) that such discrete-time tracking systems are governed on T_T by state and output equations of the respective forms

$$\begin{bmatrix} z\{(k+1)T\} \\ x_1\{(k+1)T\} \\ x_2\{(k+1)T\} \end{bmatrix} = \begin{bmatrix} I_\ell , & -TC_1 , & -TC_2 \\ f\Psi_1K_1, & \Phi_{11}-f\Psi_1K_oC_1, & \Phi_{12}-f\Psi_1K_oC_2 \\ f\Psi_2K_1, & \Phi_{21}-f\Psi_2K_oC_1, & \Phi_{22}-f\Psi_2K_oC_2 \end{bmatrix} \begin{bmatrix} z(kT) \\ x_1(kT) \\ x_2(kT) \end{bmatrix} + \begin{bmatrix} TI_\ell \\ f\Psi_1K_o \\ f\Psi_2K_o \end{bmatrix} v(kT) \tag{3.5}$$

and

$$y(kT) = [0 \ , \ C_1 \ , \ C_2] \begin{bmatrix} z(kT) \\ x_1(kT) \\ x_2(kT) \end{bmatrix} , \tag{3.6}$$

where

$$\begin{bmatrix} \Phi_{11} , \Phi_{12} \\ \Phi_{21} , \Phi_{22} \end{bmatrix} = \exp\left\{ \begin{bmatrix} A_{11} , A_{12} \\ A_{21} , A_{22} \end{bmatrix} T \right\} \tag{3.7}$$

and

$$\begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix} = \int_0^T \exp\left\{ \begin{bmatrix} A_{11} , A_{12} \\ A_{21} , A_{22} \end{bmatrix} t \right\} \begin{bmatrix} 0 \\ B_2 \end{bmatrix} dt \quad . \tag{3.8}$$

The transfer function matrix relating the plant output vector to the command input vector of the closed-loop discrete-time tracking system governed by equations (3.5) and (3.6) is clearly

$G(\lambda)$

$$= [0, C_1, C_2] \begin{bmatrix} \lambda I_\ell - I_\ell , & TC_1 & , & TC_2 \\ -f\Psi_1 K_1 , & \lambda I_{n-\ell} - \Phi_{11} + f\Psi_1 K_0 C_1 , & & -\Phi_{12} + f\Psi_1 K_0 C_2 \\ -f\Psi_2 K_1 , & -\Phi_{21} + f\Psi_2 K_0 C_1 & , & \lambda I_\ell - \Phi_{22} + f\Psi_2 K_0 C_2 \end{bmatrix}^{-1} \begin{bmatrix} TI_\ell \\ f\Psi_1 K_0 \\ f\Psi_2 K_0 \end{bmatrix} \tag{3.9}$$

and the fast-sampling tracking characteristics of this system can accordingly be elucidated by invoking the results of Porter and Shenton (1975) from the singular perturbation analysis of transfer function matrices.

These results yield the asymptotic form of $G(\lambda)$ as the sampling frequency $f \to \infty$ and thus greatly facilitate the determination of controller matrices K_0 and K_1 such that the discrete-time tracking behaviour of the closed-loop system becomes increasingly 'tight' and non-interacting as f is increased. The frequency-response and step-response characteristics of a discrete-time tracking system incorporating an open-loop unstable chemical reactor (MacFarlane and Kouvaritakis 1977) are presented in order to illustrate these general results.

3.2 Analysis

It is evident from equation (3.9) that, by regarding $\varepsilon = 1/f$ as the perturbation parameter, the asymptotic form of the transfer function matrix $G(\lambda)$ of the discrete-time tracking system as $f \to \infty$ can be determined by invoking the results of Porter and Shenton (1975) from the singular perturbation analysis of transfer function matrices. Indeed, since it follows from equations (3.7) and (3.8) that

$$\lim_{f\to\infty} f\begin{bmatrix}\Phi_{11}-I_{n-\ell} , & \Phi_{12} \\ \Phi_{21} , & \Phi_{22}-I_{\ell}\end{bmatrix} = \begin{bmatrix}A_{11} , & A_{12} \\ A_{21} , & A_{22}\end{bmatrix} \tag{3.10}$$

and

$$\lim_{f\to\infty} f\begin{bmatrix}\Psi_1 \\ \Psi_2\end{bmatrix} = \begin{bmatrix}0 \\ B_2\end{bmatrix} \quad , \tag{3.11}$$

these results indicate that as $f \to \infty$ the transfer function matrix $G(\lambda)$ assumes the asymptotic form

$$\Gamma(\lambda) = \tilde{\Gamma}(\lambda) + \hat{\Gamma}(\lambda) \tag{3.12}$$

where

$$\tilde{\Gamma}(\lambda) = C_o(\lambda I_n - I_n - TA_o)^{-1}TB_o \quad , \tag{3.13}$$

$$\hat{\Gamma}(\lambda) = C_2(\lambda I_\ell - I_\ell - A_4)^{-1}B_2K_o \quad , \tag{3.14}$$

$$A_o = \begin{bmatrix} -K_o^{-1}K_1 & , & 0 \\ A_{12}C_2^{-1}K_o^{-1}K_1 & , & A_{11}-A_{12}C_2^{-1}C_1 \end{bmatrix} \quad , \tag{3.15}$$

$$B_o = \begin{bmatrix} 0 \\ A_{12}C_2^{-1} \end{bmatrix} \quad , \tag{3.16}$$

$$C_o = [K_o^{-1}K_1 \ , \ 0] \quad , \tag{3.17}$$

and

$$A_4 = -B_2K_oC_2 \quad . \tag{3.18}$$

It follows from equations (3.12),(3.13), and (3.15) that the 'slow' modes Z_s of the tracking system correspond as $f \to \infty$ to the poles $Z_1 \cup Z_2$ of $\tilde{\Gamma}(\lambda)$ where

$$Z_1 = \{\lambda \in C: |\lambda I_\ell - I_\ell + TK_o^{-1}K_1| = 0\} \tag{3.19}$$

and

$$Z_2 = \{\lambda \in C: |\lambda I_{n-\ell} - I_{n-\ell} - TA_{11} + TA_{12}C_2^{-1}C_1| = 0\} \quad , \tag{3.20}$$

and from equations (3.12),(3.14), and (3.18) that the 'fast' modes Z_f of the tracking system correspond as $f \to \infty$ to the poles Z_3 of $\hat{\Gamma}(\lambda)$ where

$$Z_3 = \{\lambda \in C: |\lambda I_\ell - I_\ell + C_2B_2K_o| = 0\} \quad . \tag{3.21}$$

Furthermore, it follows from equations (3.13),(3.15),(3.16), and(3.17) that the 'slow' transfer function matrix

$$\tilde{\Gamma}(\lambda) = 0 \tag{3.22}$$

and from equations (3.14) and (3.18) that the 'fast' transfer function matrix

$$\hat{\Gamma}(\lambda) = (\lambda I_\ell - I_\ell + C_2 B_2 K_o)^{-1} C_2 B_2 K_o \quad . \tag{3.23}$$

Hence, in view of equations (3.22) and (3.23), it is evident from equation (3.12) that as $f \to \infty$ the transfer function matrix $G(\lambda)$ of the discrete-time tracking system assumes the asymptotic form

$$\Gamma(\lambda) = (\lambda I_\ell - I_\ell + C_2 B_2 K_o)^{-1} C_2 B_2 K_o \tag{3.24}$$

in consonance with the fact that only the 'fast' modes corresponding to the poles Z_3 remain both controllable and observable as $f \to \infty$. Indeed, the 'slow' transfer function matrix $\tilde{\Gamma}(\lambda)$ vanishes precisely because the 'slow' modes corresponding to the poles Z_1 become asymptotically uncontrollable as $f \to \infty$ in view of the block structure of the matrices A_o and B_o in equations (3.15) and (3.16) whilst the 'slow' modes corresponding to the poles Z_2 become asymptotically unobservable as $f \to \infty$ in view of the block structure of the matrices A_o and C_o in equations (3.15) and (3..7).

3.3 Synthesis

It is evident from equations (3.5) and (3.6) that tracking will occur in the sense of equation (3.4) provided only that

$$Z_s \cup Z_f \subset \mathcal{D}^- \tag{3.25}$$

where $\mathcal{D}^-$ is the open unit disc. In view of equations (3.19)

(3.20),and(3.21),the 'slow' and 'fast' modes will satisfy the tracking requirement (3.25) for sufficiently fast sampling frequencies if the controller matrices K_o and K_1 are chosen such that both $Z_1 \subset \mathcal{D}^-$ for sufficiently small sampling periods and $Z_3 \subset \mathcal{D}^-$ in the case of minimum-phase plants for which (Porter and D'Azzo 1977) the set of transmission zeros

$$Z_t = \{\lambda \in \mathbb{C} : |\lambda I_{n-\ell} - A_{11} + A_{12} C_2^{-1} C_1| = 0\} \subset \mathbb{C}^- \tag{3.26}$$

where $\mathbb{C}^-$ is the open left half-plane since it is then immediately obvious from equation (3.20) that $Z_2 \subset \mathcal{D}^-$ for sufficiently small sampling periods. Moreover, in such cases, tracking will become increasingly 'tight' as $f \to \infty$ in view of equation (3.24). Furthermore, if K_o is chosen such that

$$C_2 B_2 K_o = \mathrm{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_\ell\} \tag{3.27}$$

where $1-\sigma_j \in \mathbb{R} \cap \mathcal{D}^-$ $(j=1,2,\ldots,\ell)$, it follows from equation (3.24) that the transfer function matrix $G(\lambda)$ of the discrete-time tracking system will assume the diagonal asymptotic form

$$\Gamma(\lambda) = \mathrm{diag}\left\{\frac{\sigma_1}{\lambda-1+\sigma_1}, \frac{\sigma_2}{\lambda-1+\sigma_2}, \ldots, \frac{\sigma_\ell}{\lambda-1+\sigma_\ell}\right\} \tag{3.28}$$

and therefore that increasingly non-interacting discrete-time tracking behaviour will occur as $f \to \infty$.

3.4 Illustrative Example

These general results can be conveniently illustrated by designing a fast-sampling error-actuated digital controller for an open-loop unstable chemical reactor governed on T

by the respective state and output equations (MacFarlane and Kouvaritakis 1977)

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix} = \begin{bmatrix} 1.38 & , -0.2077 & , \ 6.715 & , -5.676 \\ -0.5814 & , -4.29 & , \ 0 & , \ 0.675 \\ 1.067 & , \ 4.273 & , -6.654 & , \ 5.893 \\ 0.048 & , \ 4.273 & , \ 1.343 & , -2.104 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & , \ 0 \\ 5.679 & , \ 0 \\ 1.136 & , -3.146 \\ 1.136 & , \ 0 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \tag{3.29}$$

and

$$y(t) = \begin{bmatrix} 1 , 0 , 1 , -1 \\ 0 , 1 , 0 , 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} \tag{3.30}$$

from which it can be readily verified that $Z_t = \{-1.192, -5.039\} \subset C^-$ and that the first Markov parameter

$$C_2B_2 = \begin{bmatrix} 0 & , -3.146 \\ 5.679 & , \ 0 \end{bmatrix} . \tag{3.31}$$

In case $\{\sigma_1, \sigma_2\} = \{1,1\}$ and $K_1 = 2K_0$, it follows from equations (3.3), (3.27), and (2.38) that the corresponding fast-sampling error-actuated digital controller is governed on T_T by the control-law equation

$$\begin{bmatrix} u_1(kT) \\ u_2(kT) \end{bmatrix} = f\left\{\begin{bmatrix} 0 & ,0.1761 \\ -0.3179, & 0 \end{bmatrix}\begin{bmatrix} e_1(kT) \\ e_2(kT) \end{bmatrix} + \begin{bmatrix} 0 & ,0.3522 \\ 0.6358, & 0 \end{bmatrix}\begin{bmatrix} z_1(kT) \\ z_2(kT) \end{bmatrix}\right\} \tag{3.32}$$

and it is evident from equations (3.19), (3.20), and (3.21) that $Z_1 = \{1-2T, 1-2T\}$, $Z_2 = \{1-1.192T, 1-5.039T\}$, and $Z_3 = \{0,0\}$. It is also evident from equation (3.24) that the asymptotic transfer function matrix assumes the diagonal form

$$\Gamma(\lambda) = \begin{bmatrix} \frac{1}{\lambda} & , 0 \\ 0 & , \frac{1}{\lambda} \end{bmatrix} \tag{3.33}$$

and therefore that the closed-loop discrete-time tracking system incorporating the chemical reactor will exhibit increasingly 'tight' and non-interacting tracking behaviour as $f \to \infty$ when the piecewise-constant control input vector $[u_1(t) \ , \ u_2(t)]^T = [u_1(kT) \ , \ u_2(kT)]^T$, $t \in [kT \ , \ (k+1)T)$, $kT \in T_T$, is generated by the fast-sampling digital controller governed on T_T by equation (3.32).

The actual frequency-response loci $G(e^{i\omega T})$ for $\omega T \in [0 \ , \ 2\pi]$ are shown in Figs 3.1, 3.2, and 3.3 when $T = 0.04$, 0.02, and 0.01, respectively, and it is clear that the actual frequency-response loci approach the asymptotic frequency-response loci $\Gamma(e^{i\omega T})$ as the sampling frequency f is increased. The corresponding step-response characteristics are shown in Fig 3.4 and Fig 3.5 when $[v_1(t) \ , \ v_2(t)]^T = [1 \ , \ 0]^T$ and $[v_1(t) \ , \ v_2(t)] = [0 \ , \ 1]^T$, respectively, and it is evident that increasingly 'tight' and non-interacting tracking occurs as the sampling frequency f is increased.

3.5 CONCLUSION

Singular perturbation methods have been used to exhibit the asymptotic structure of the transfer function matrices of discrete-time tracking systems incorporating linear multivariable plants which are amenable to fast-sampling error-actuated digital control. It has been shown that these results greatly facilitate the determination of controller matrices which ensure that the closed-loop behaviour of such discrete-time tracking systems become increasingly 'tight' and non-interacting as the sampling frequency f is increased. These general results have been illustrated by the presentation of the frequency-response and step-response characteristics of a discrete-time tracking system incorporating an open-loop unstable chemical reactor.

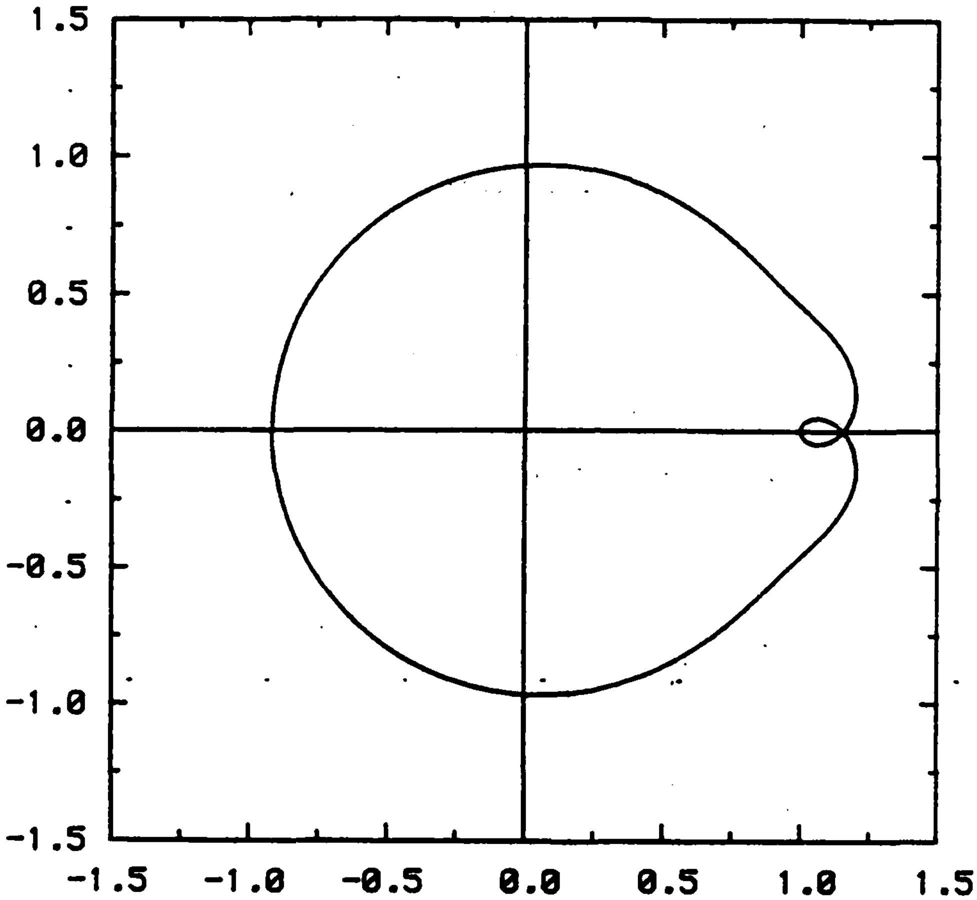


Fig 3.1(a): $G_{11}(e^{i\omega T})$; T = 0.04

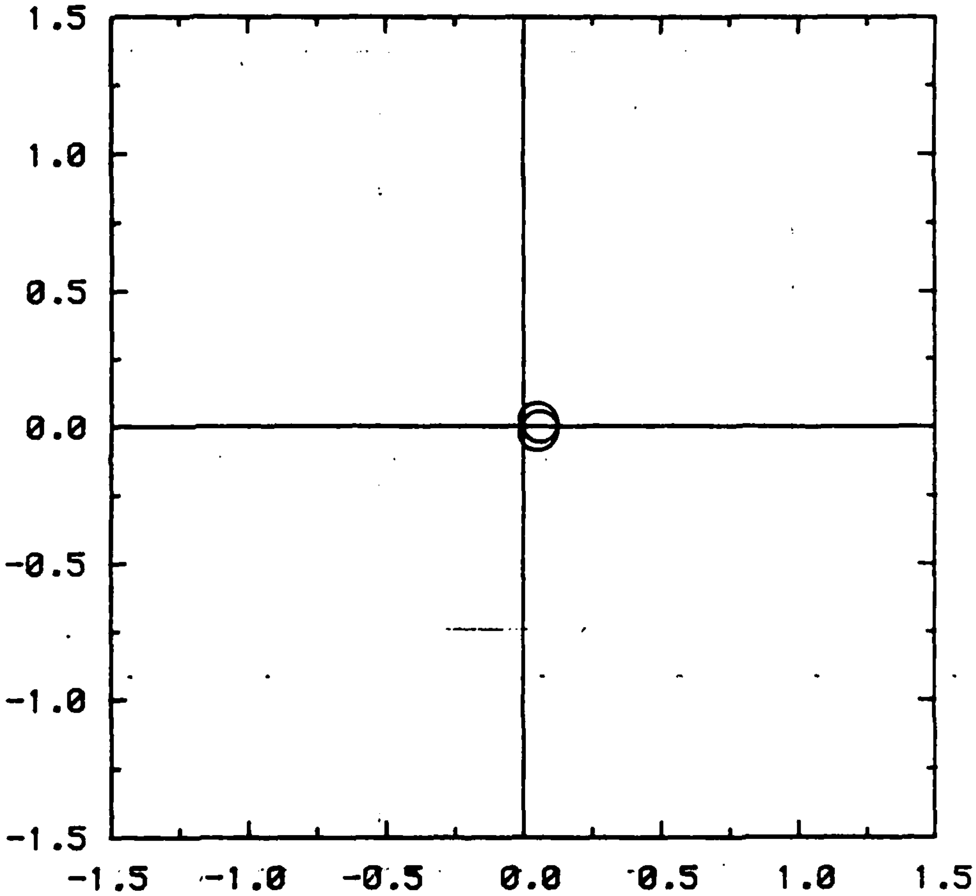


Fig 3.1(b): $G_{12}(e^{i\omega T})$; T = 0.04

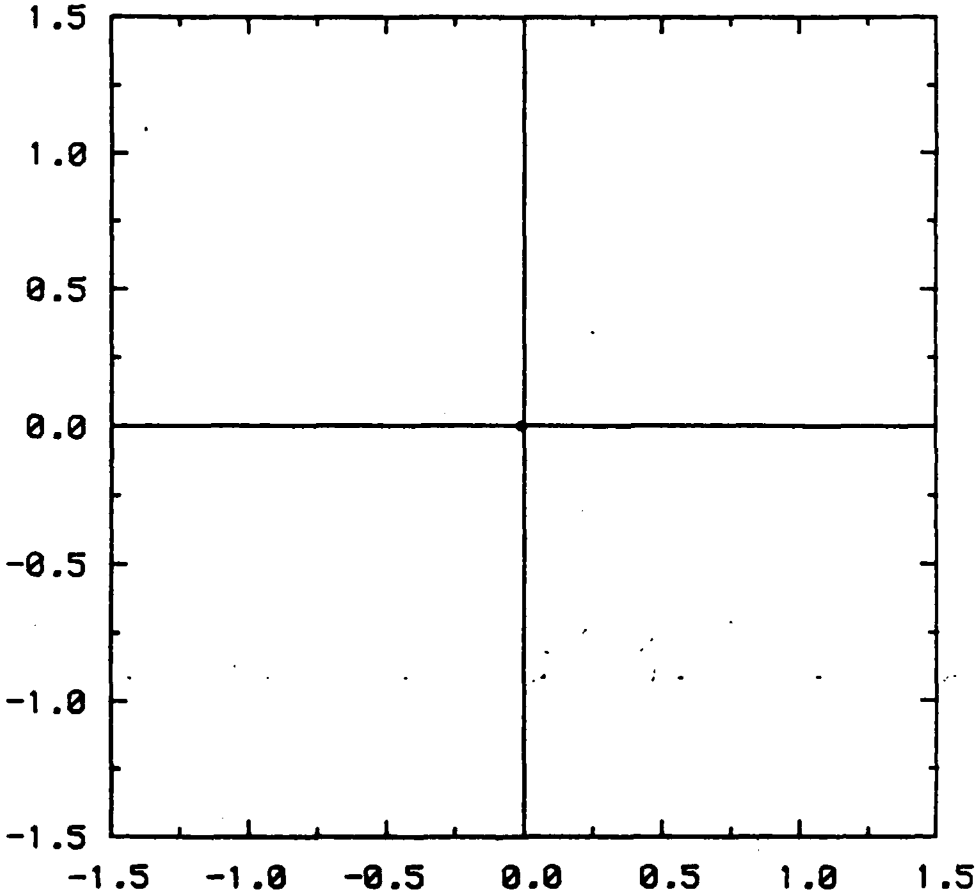


Fig 3.1(c): $G_{21}(e^{i\omega T})$; T = 0.04

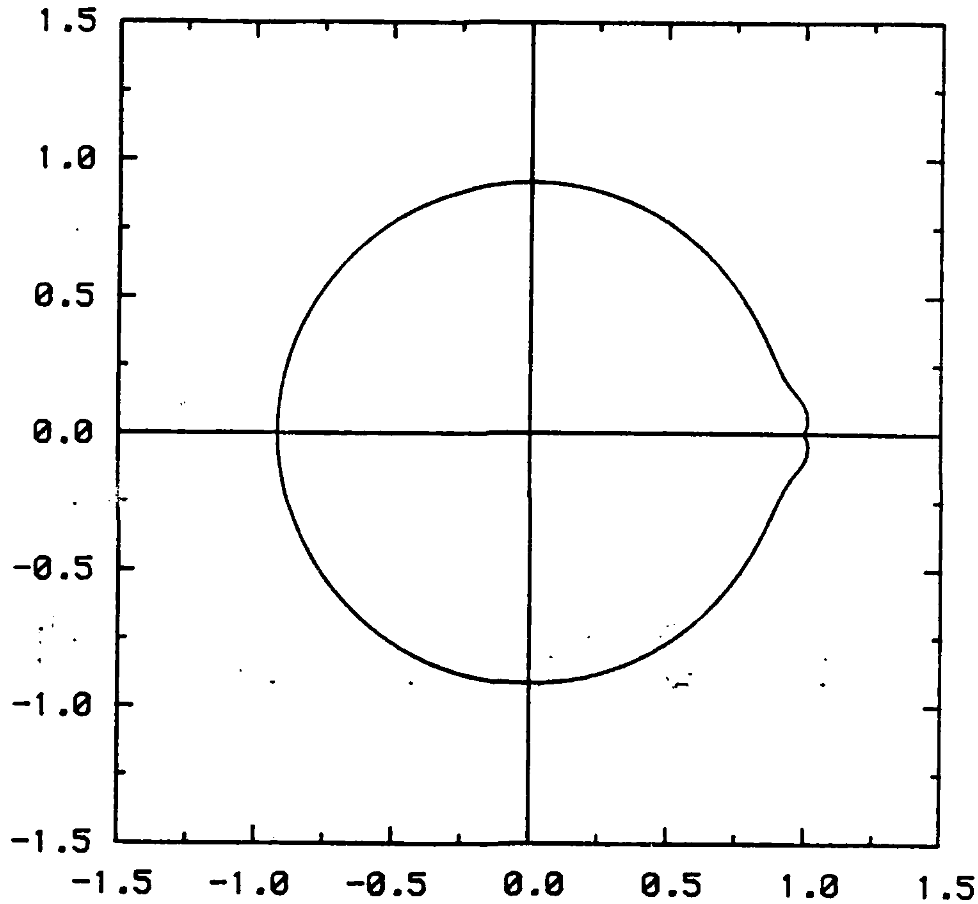


Fig 3.1(d): $G_{22}(e^{i\omega T})$; T = 0.04

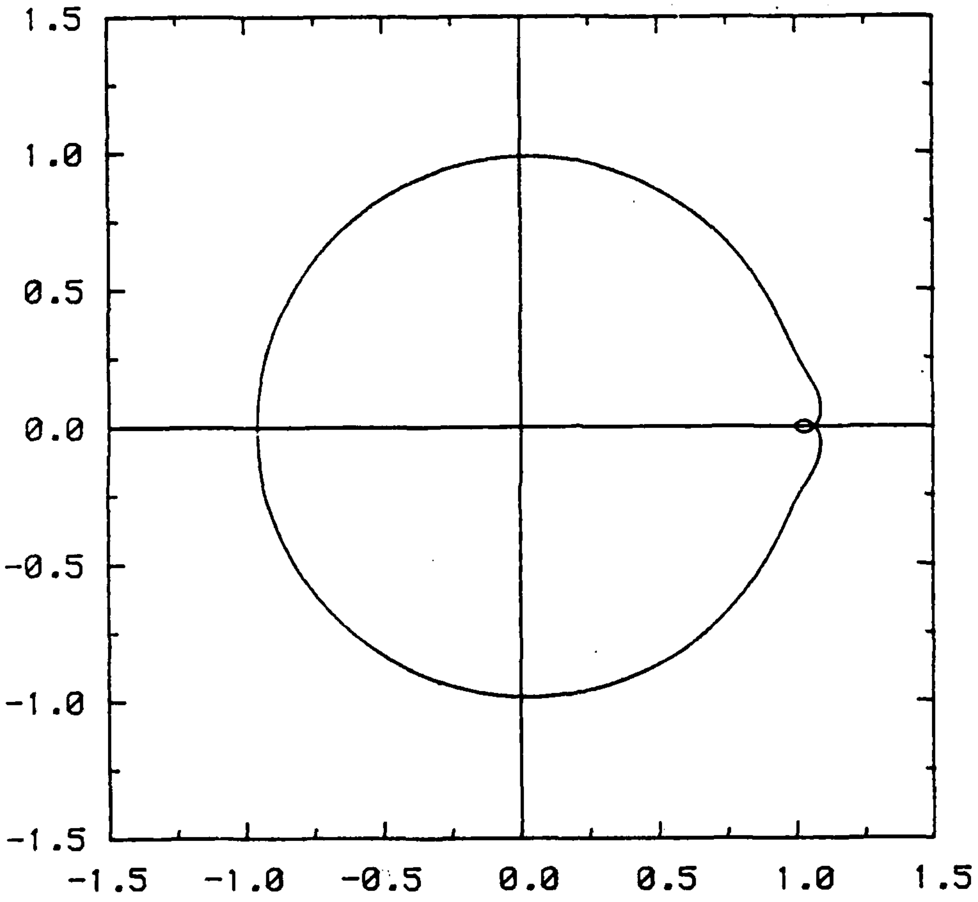


Fig 3.2(a): $G_{11}(e^{i\omega T})$; T = 0.02

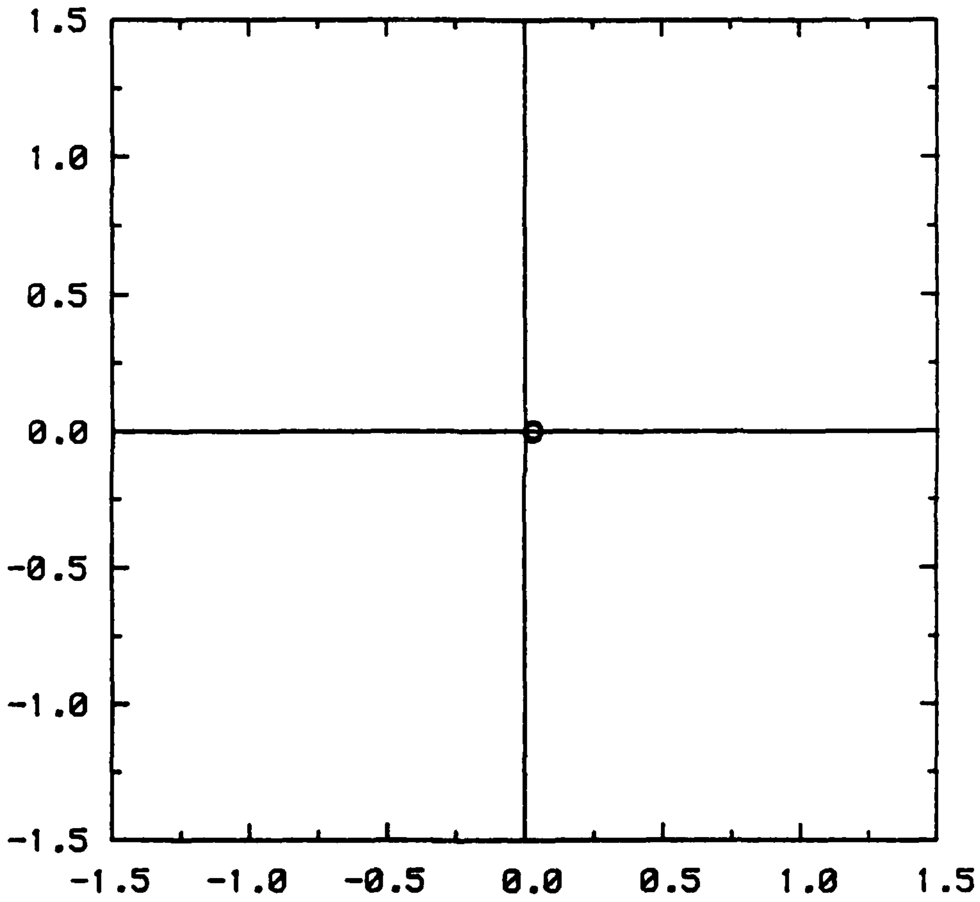


Fig 3.2(b): $G_{12}(e^{i\omega T})$; T = 0.02

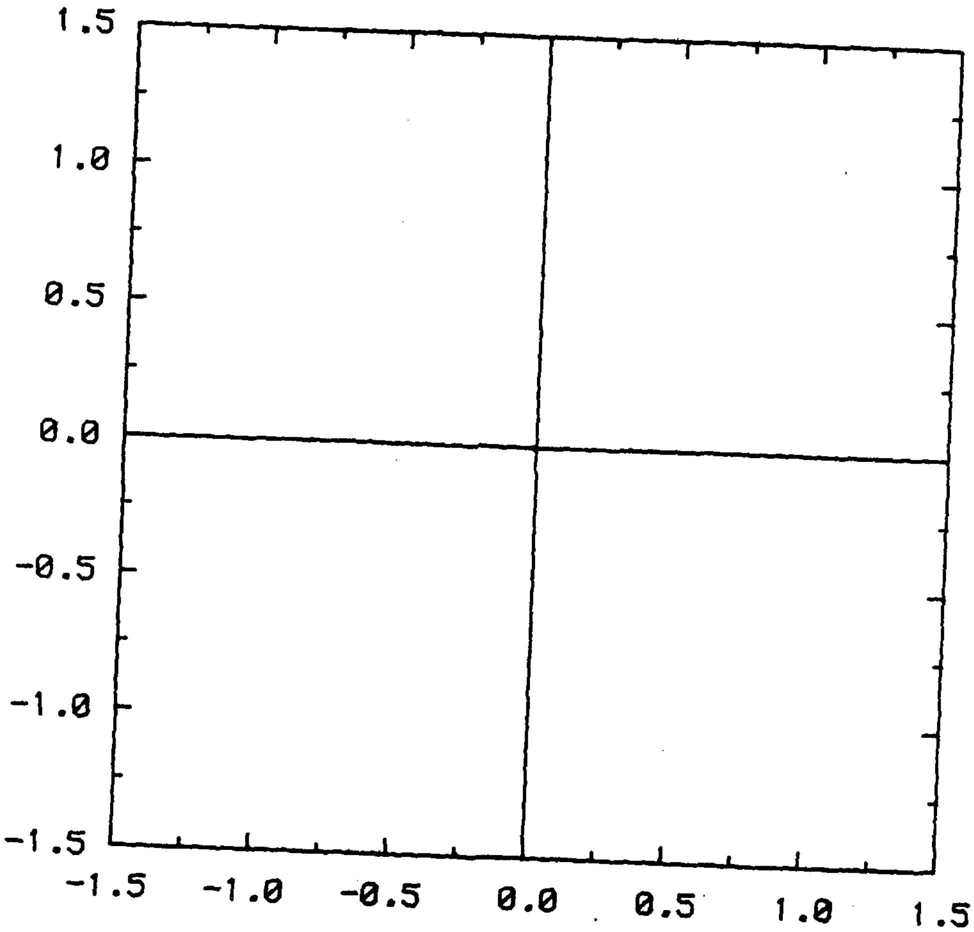


Fig 3.2(c): $G_{21}(e^{i\omega T})$; T = 0.02

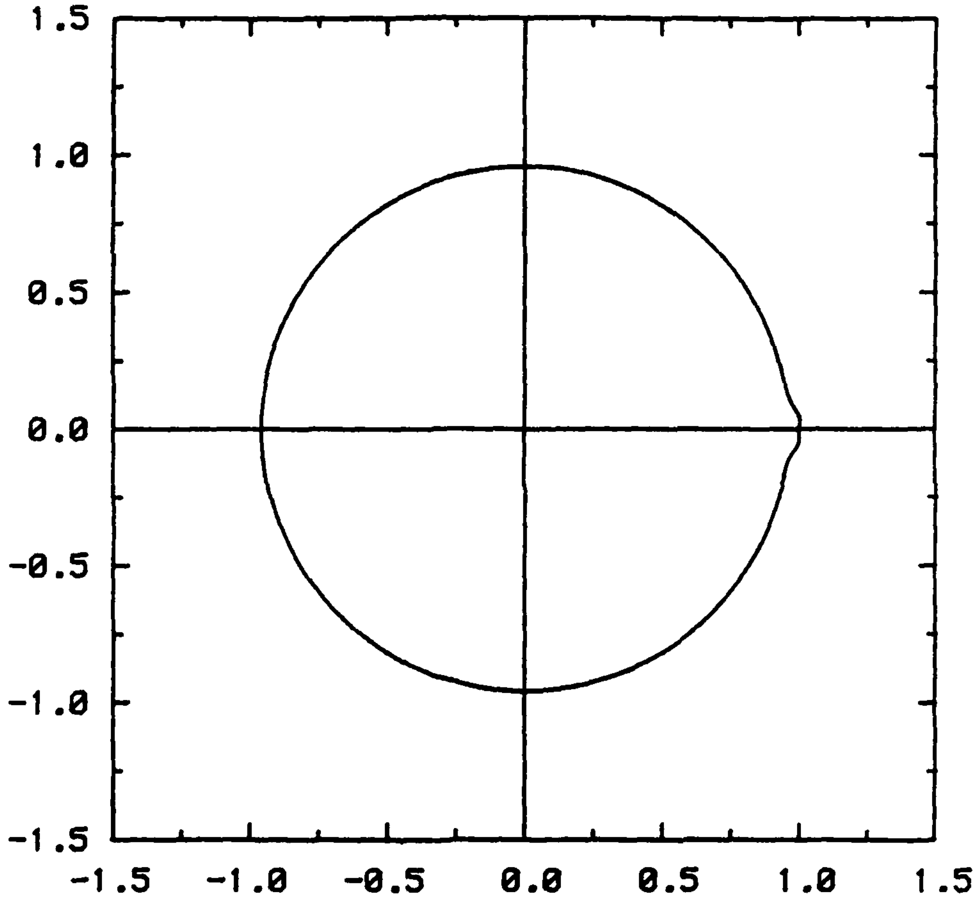


Fig 3.2(d): $G_{22}(e^{i\omega T})$; T = 0.02

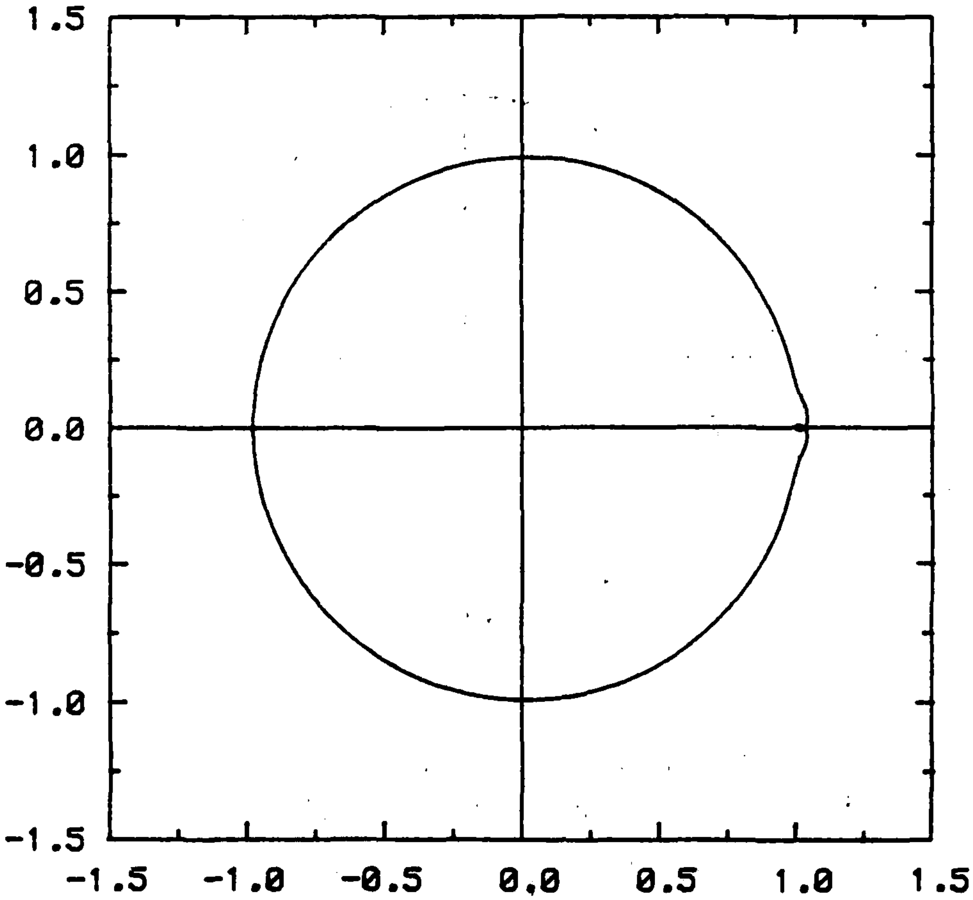


Fig 3.3(a): $G_{11}(e^{i\omega T})$; T = 0.01

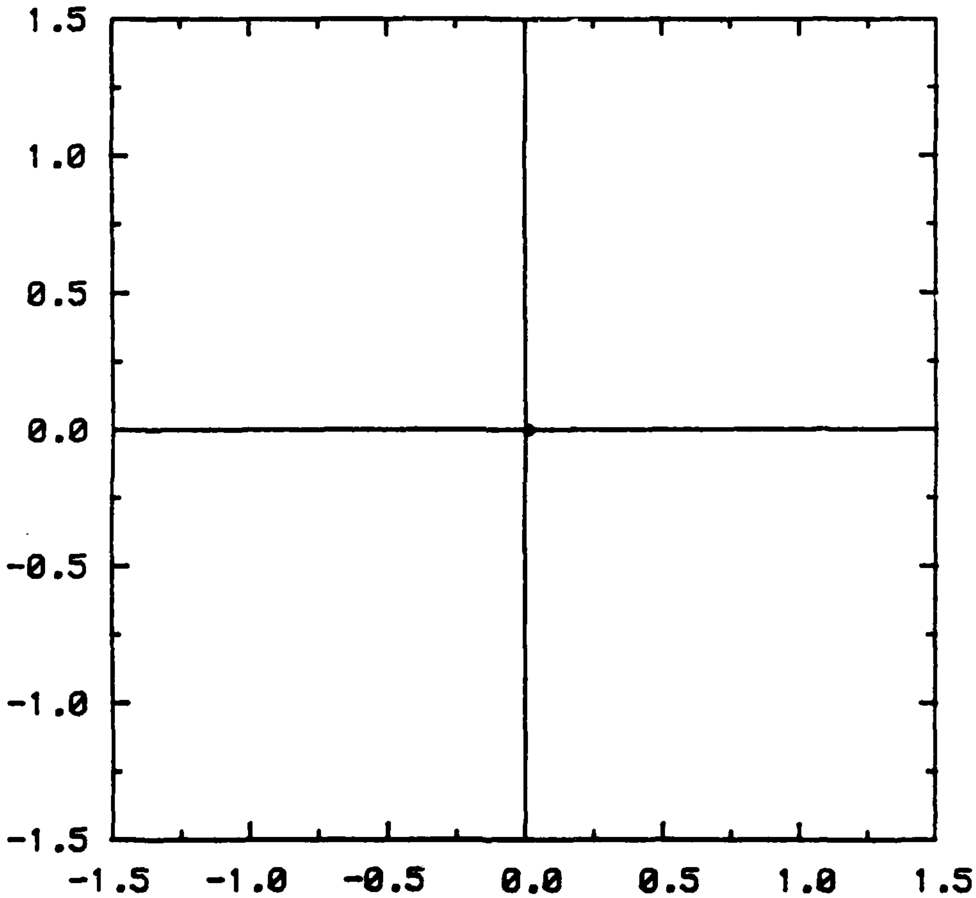


Fig 3.3(b): $G_{12}(e^{i\omega T})$; T = 0.01

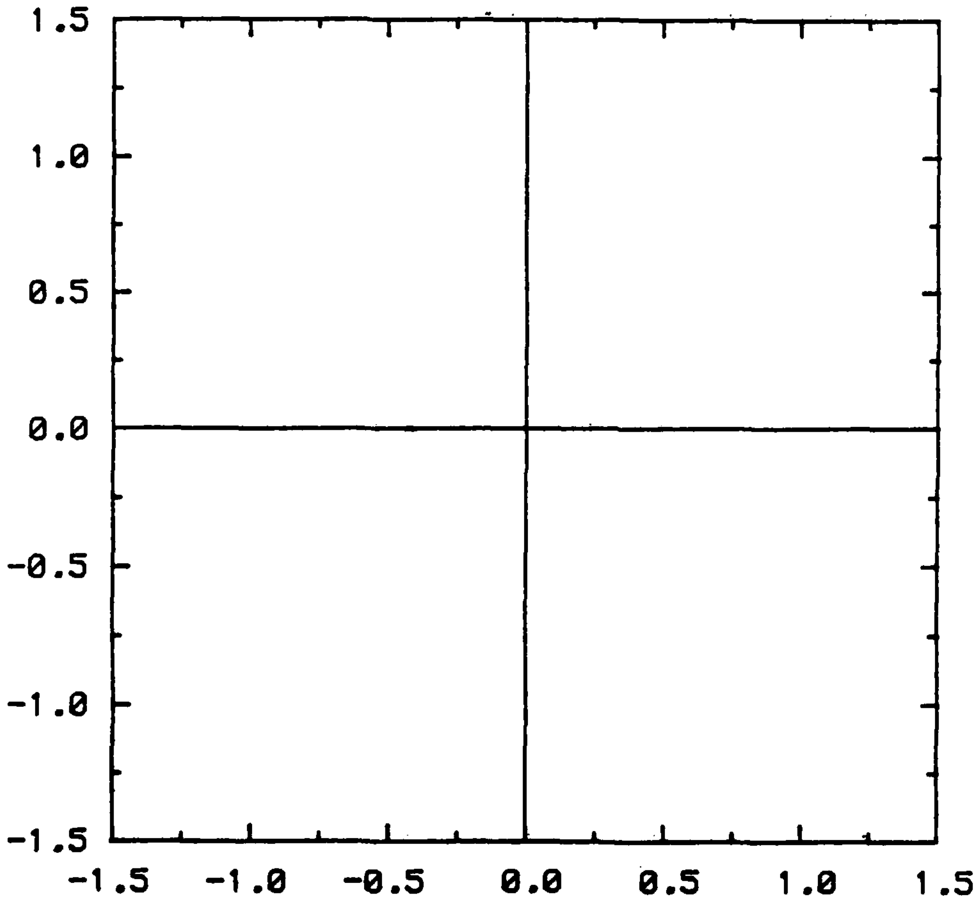


Fig 3.3(c): $G_{21}(e^{i\omega T})$; T = 0.01

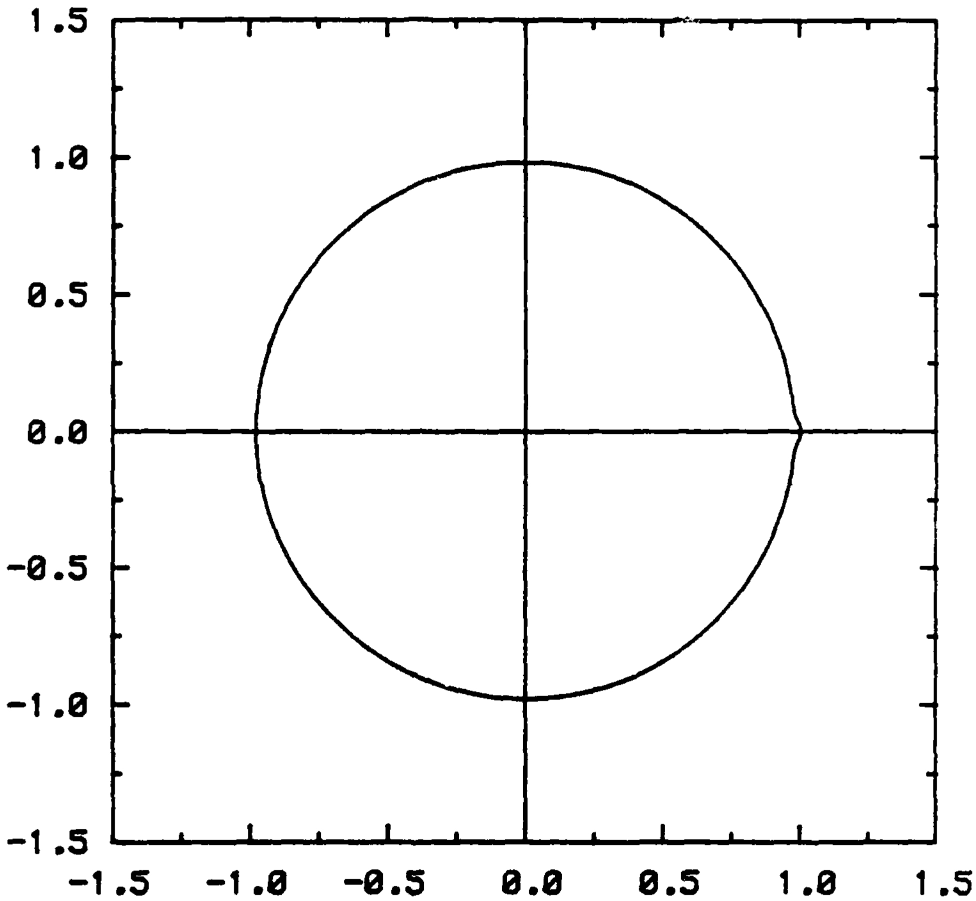


Fig 3.3(d): $G_{22}(e^{i\omega T})$; T = 0.01

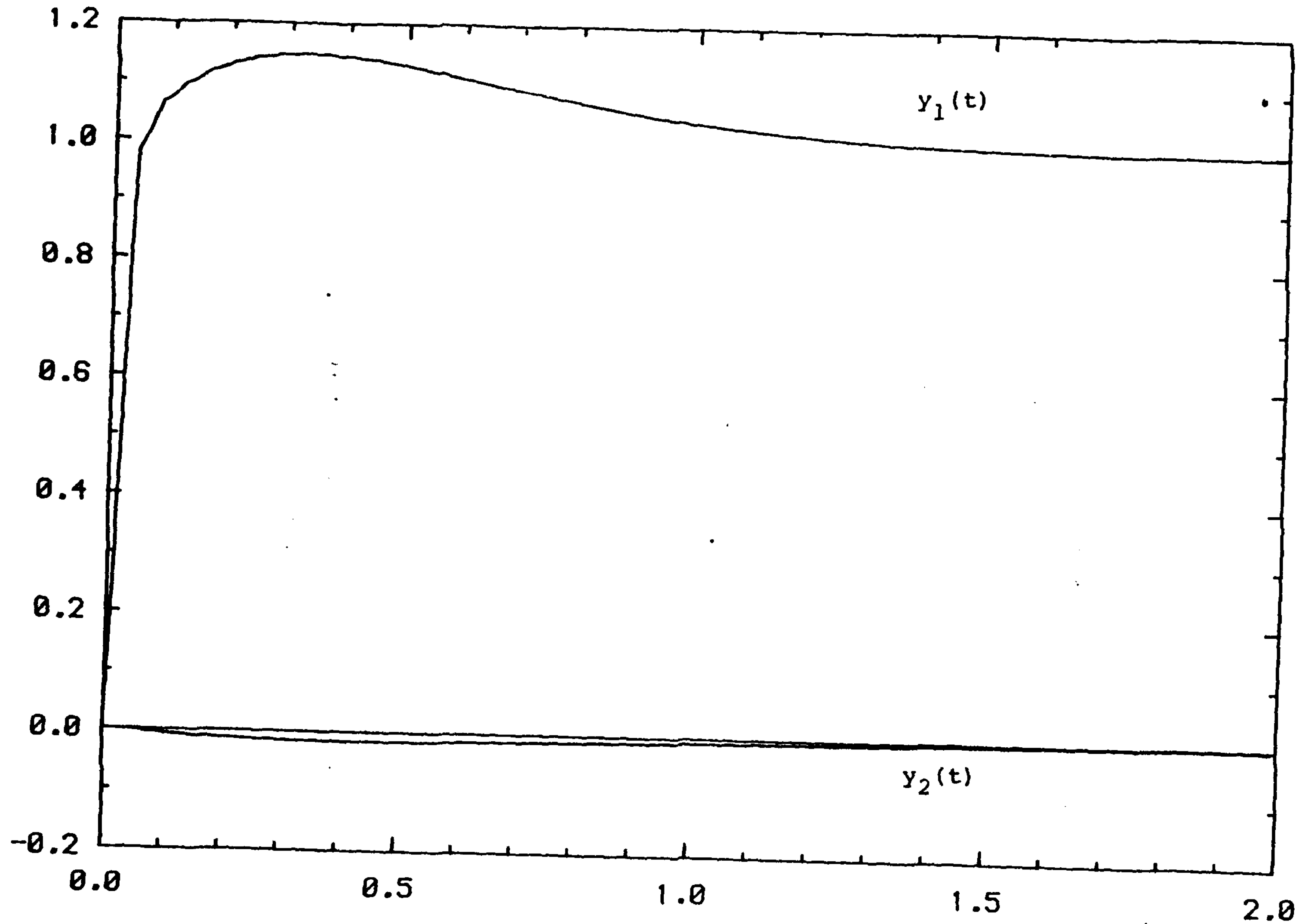


Fig 3.4(a): T = 0.04

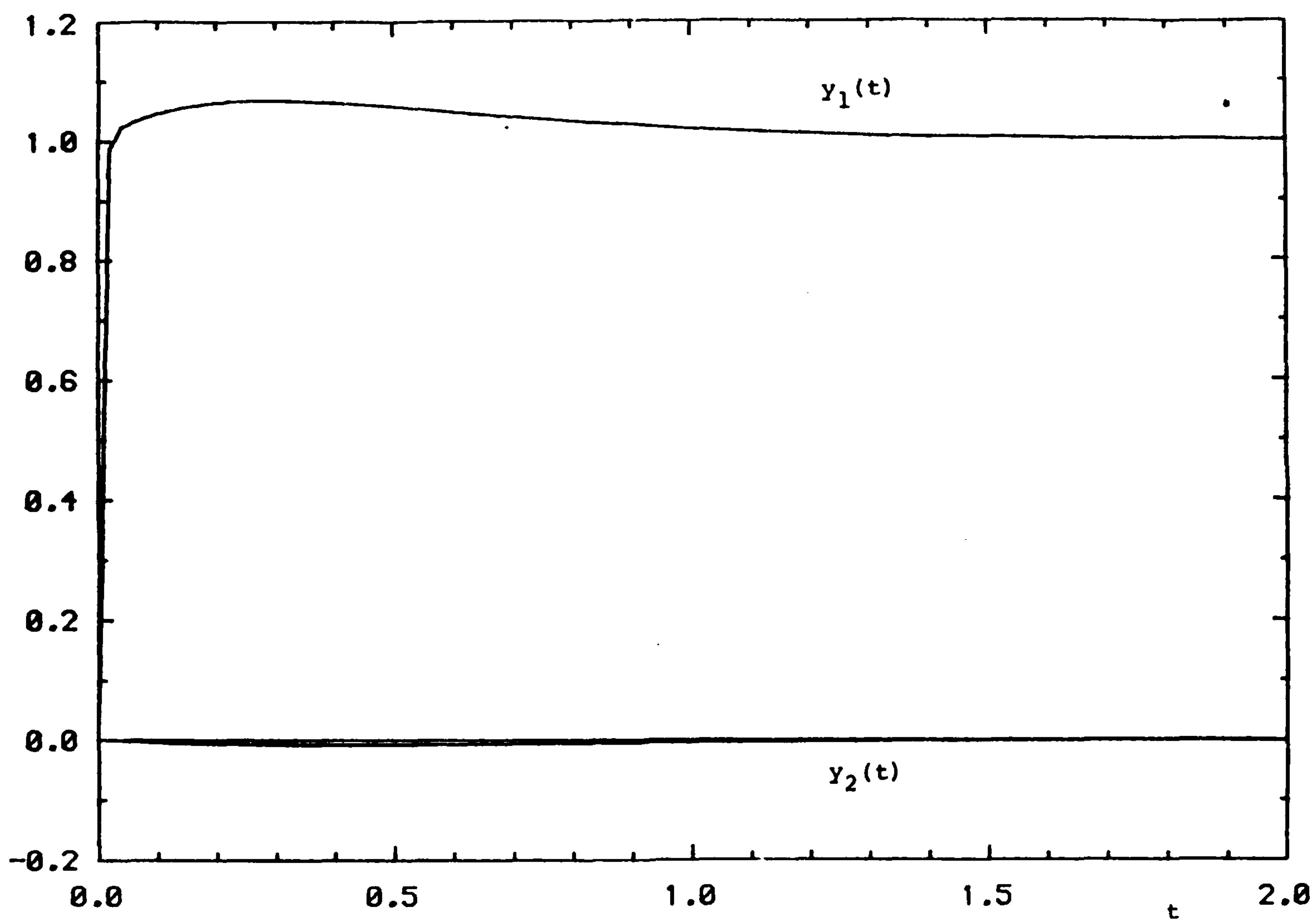


Fig 3.4(b): T = 0.02

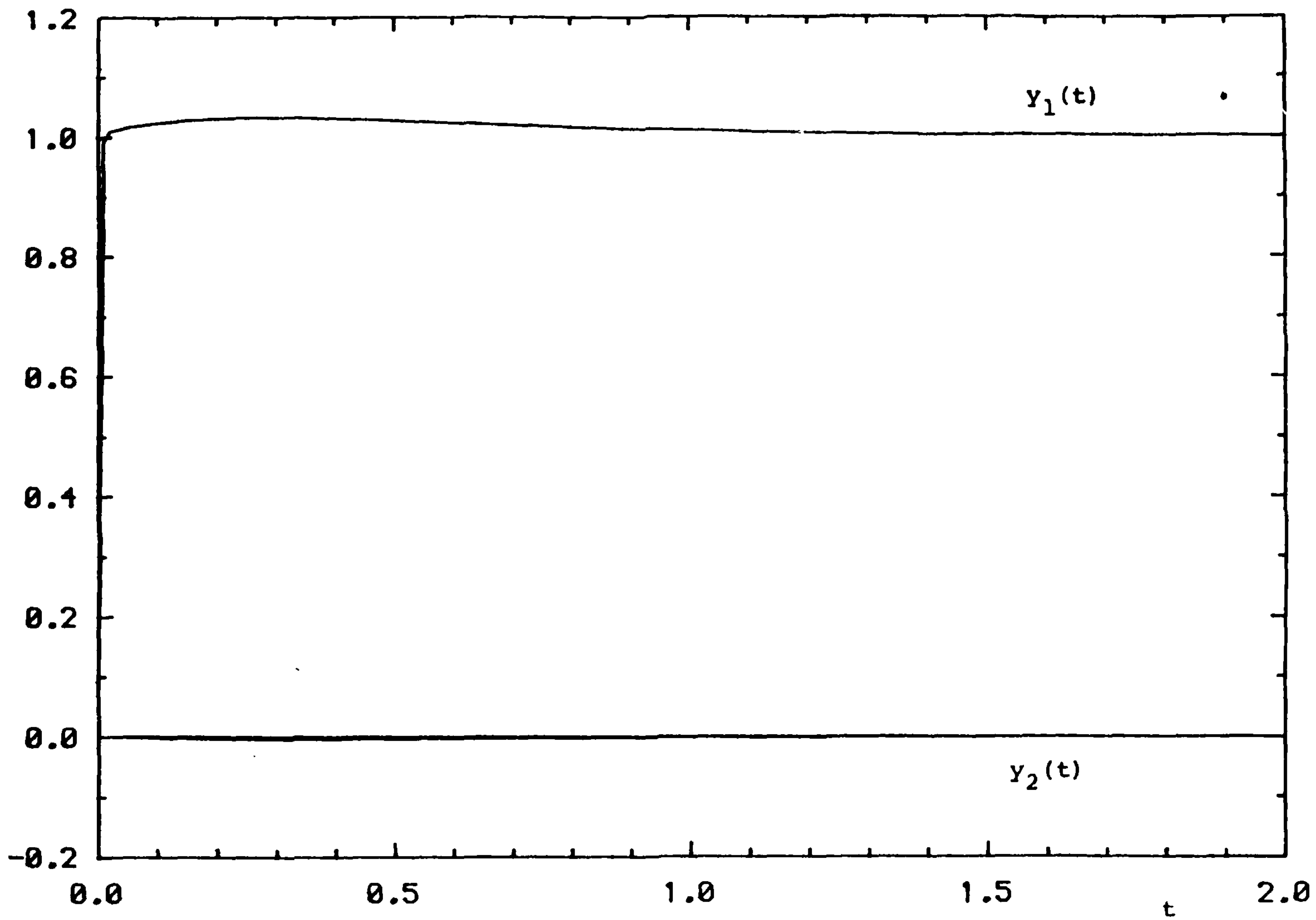


Fig 3.4(c): T = 0.01

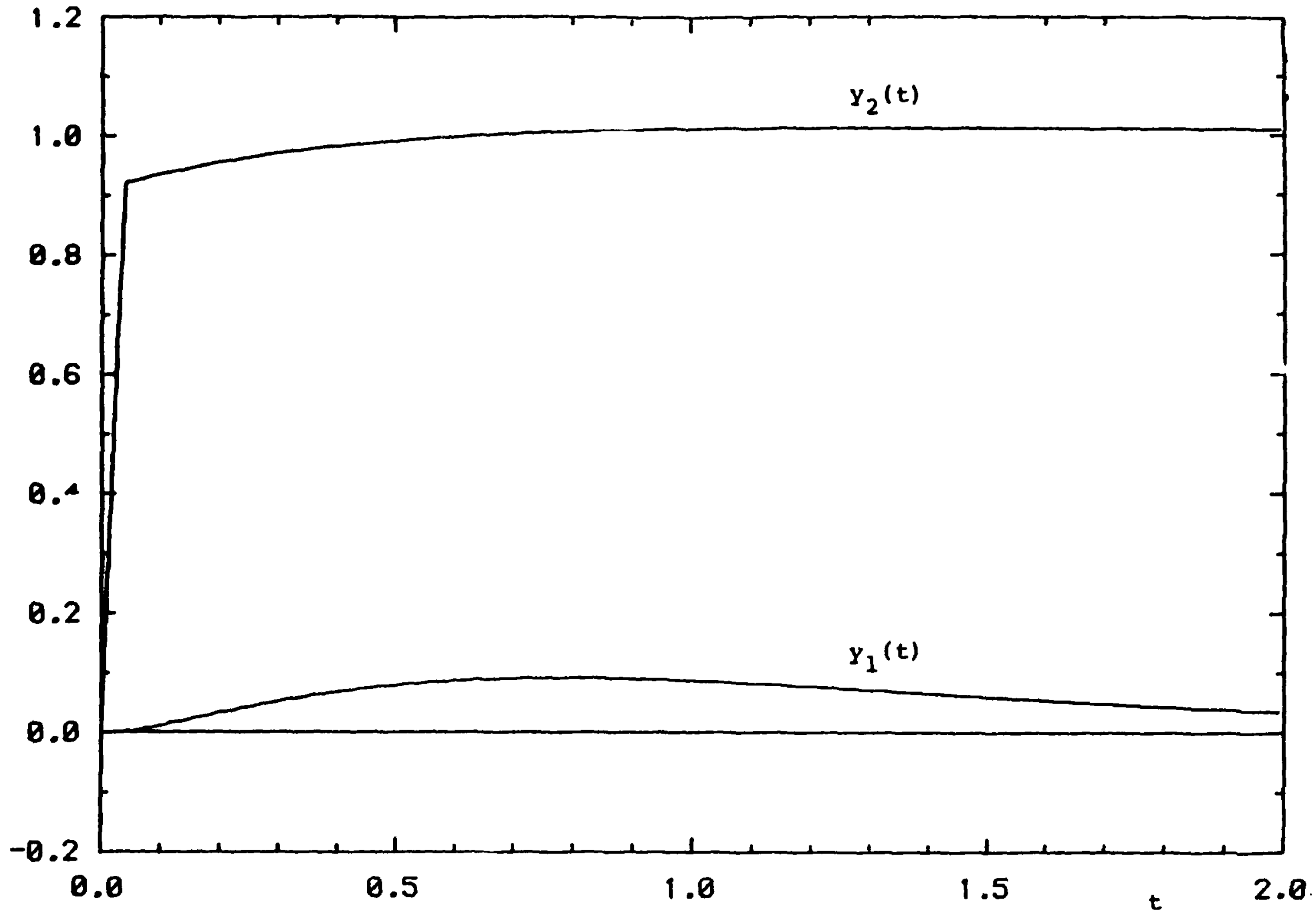


Fig 3.5(a): T = 0.04

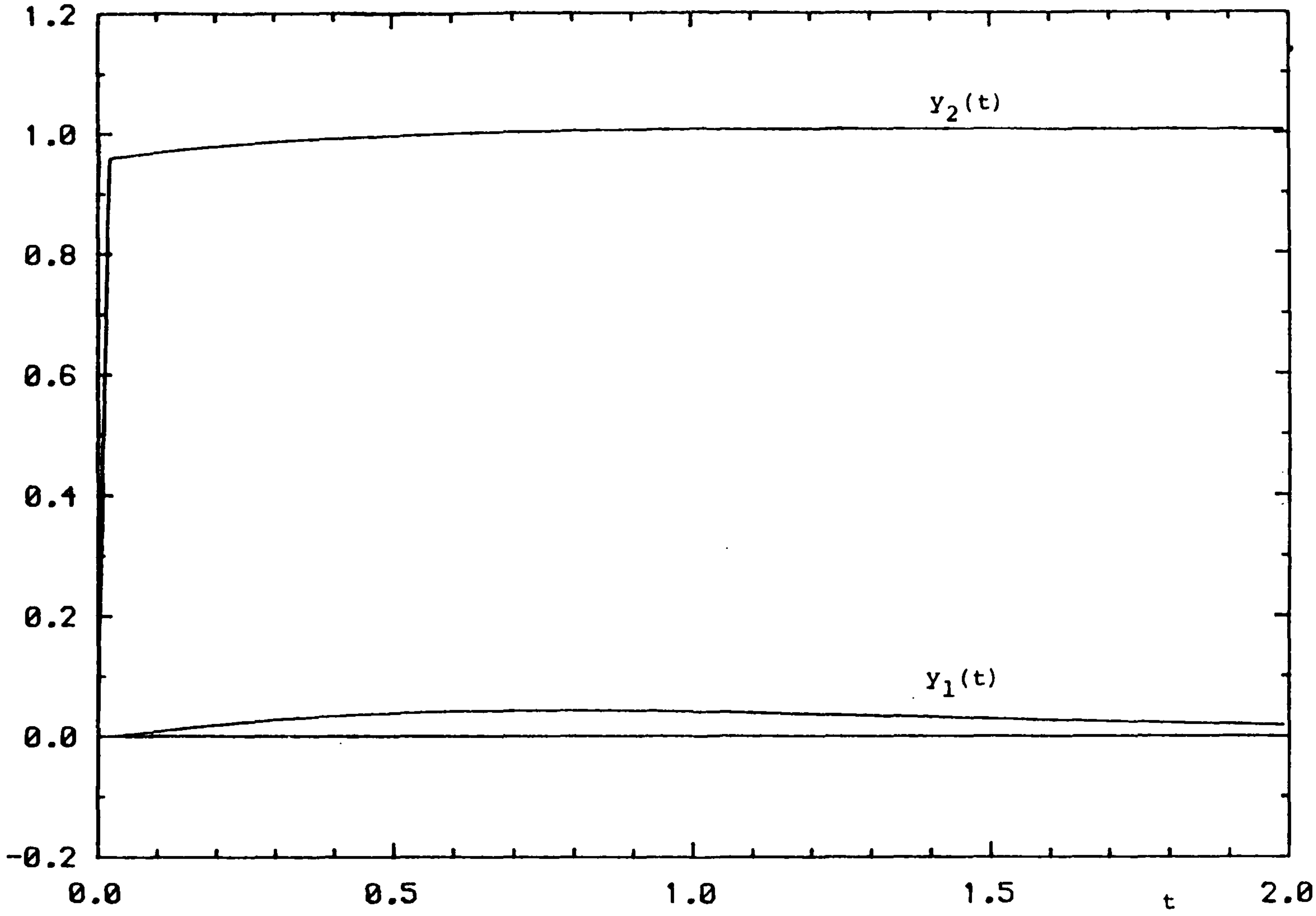


Fig 3.5(b): T = 0.02

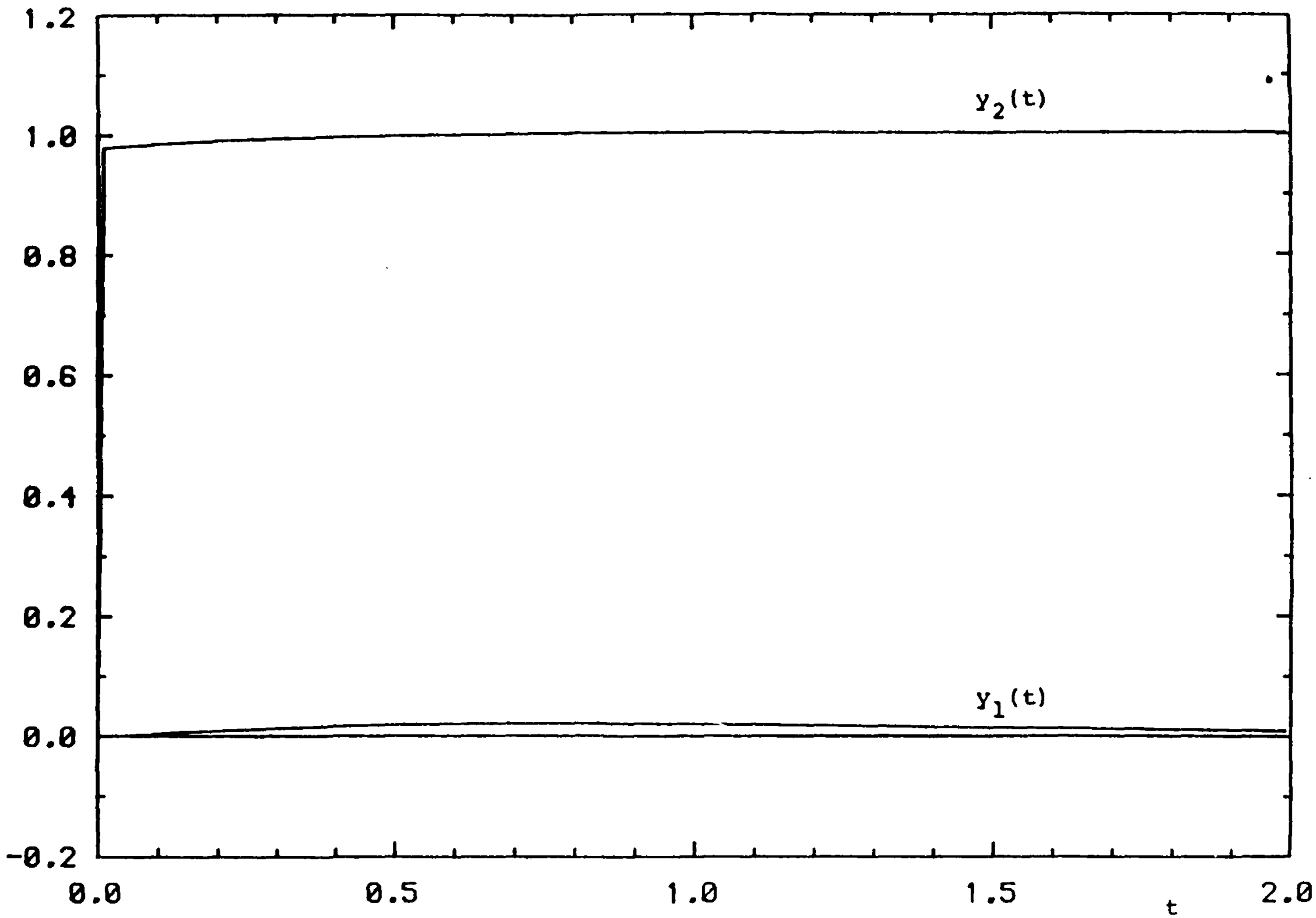


Fig 3.5(c): T = 0.01

CHAPTER 4

DESIGN OF TRACKING SYSTEMS INCORPORATING INNER-LOOP COMPENSATORS AND HIGH-GAIN ERROR-ACTUATED CONTROLLERS

4.1 Introduction

In this chapter, singular perturbation methods are used to exhibit the asymptotic structure of the transfer function matrices of continuous-time tracking systems incorporating linear multivariable plants which are amenable to high-gain error-actuated analogue control (Porter and Bradshaw 1979) only if extra plant output measurements are generated by the introduction of appropriate transducers and processed by inner-loop compensators. Such tracking systems consist of a linear multivariable plant governed on the continuous-time set $T = [0,+\infty)$ by state, output, and measurement equations of the respective forms

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} , & A_{12} \\ A_{21} , & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u(t) \quad , \tag{4.1}$$

$$y(t) = [C_1 \ , \ C_2] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad , \tag{4.2}$$

and

$$w(t) = [F_1 \ , \ F_2] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad , \tag{4.3}$$

together with a high-gain error-actuated analogue controller governed on T by a control-law equation of the form

$$u(t) = g\{K_0 e(t) + K_1 z(t)\} \tag{4.4}$$

which is required to generate the control input vector $u(t)$ so as to cause the output vector $y(t)$ to track any constant command input vector $v(t)$ on T in the sense that

$$\lim_{t\to\infty}\{v(t)-y(t)\} = 0 \tag{4.5}$$

as a consequence of the fact that the error vector $e(t) = v(t) - w(t)$ assumes the steady-state value

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty}\{v(t)-w(t)\} = 0 \tag{4.6}$$

for arbitrary initial conditions. In equations (4.1), (4.2), (4.3), and (4.4), $x_1(t) \in R^{n-\ell}$, $x_2(t) \in R^{\ell}$, $u(t) \in R^{\ell}$, $y(t) \in R^{\ell}$, $w(t) \in R^{\ell}$, $A_{11} \in R^{(n-\ell)x(n-\ell)}$, $A_{12} \in R^{(n-\ell)x\ell}$, $A_{21} \in R^{\ell x(n-\ell)}$, $A_{22} \in R^{\ell x\ell}$, $B_2 \in R^{\ell x\ell}$, $C_1 \in R^{\ell x(n-\ell)}$, $C_2 \in R^{\ell x\ell}$, $F_1 \in R^{\ell x(n-\ell)}$, $F_2 \in R^{\ell x\ell}$, rank $C_2B_2 < \ell$, rank $F_2B_2 = \ell$, $e(t) \in R^{\ell}$, $v(t) \in R^{\ell}$, $z(t) = z(o) + \int_o^t edt \in R^{\ell}$, $K_o \in R^{\ell x\ell}$, $K_1 \in R^{\ell x\ell}$, $g \in R^{+}$, and

$$[F_1 \ , \ F_2] = [C_1+MA_{11} \ , \ C_2+MA_{12}] \tag{4.7}$$

where $M \in R^{\ell x(n-\ell)}$. It is evident from equations (4.2), (4.3), and (4.7) that the vector

$$w(t) - y(t) = [MA_{11} \ , \ MA_{12}]\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{4.8}$$

of extra measurements is such that $v(t)$ and $y(t)$ satisfy the tracking condition (4.5) for any $M \in R^{\ell x(n-\ell)}$ if $e(t)$ satisfies the steady-state condition (4.6) since equation (4.1) clearly implies that

$$\lim_{t\to\infty}[A_{11} \ , \ A_{12}]\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = 0 \tag{4.9}$$

in any steady state. However, the condition that rank $F_2B_2 = \ell$ requires that $C_2 \in R^{\ell x \ell}$ and $A_{12} \in R^{(n-\ell)x\ell}$ are such that $M \in R^{\ell x(n-\ell)}$ can be chosen so that

$$\text{rank } F_2 = \text{rank}(C_2 + MA_{12}) = \ell \quad . \tag{4.10}$$

It is evident from equations (4.1),(4.2),(4.3), and (4.4) that such continuous-time tracking systems are governed on T by state and output equations of the respective forms

$$\begin{bmatrix} \dot{z}(t) \\ \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & , & -F_1 & , & -F_2 \\ 0 & , & A_{11} & , & A_{12} \\ gB_2K_1 & , & A_{21}-gB_2K_oF_1 & , & A_{22}-gB_2K_oF_2 \end{bmatrix} \begin{bmatrix} z(t) \\ x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} I_\ell \\ 0 \\ gB_2K_o \end{bmatrix} v(t) \tag{4.11}$$

and

$$y(t) = [0 \ , \ C_1 \ , \ C_2] \begin{bmatrix} z(t) \\ x_1(t) \\ x_2(t) \end{bmatrix} \quad . \tag{4.12}$$

The transfer function matrix relating the plant output vector to the command input vector of the closed-loop continuous-time tracking system governed by equations (4.11) and (4.12) is clearly

$$G(\lambda) = [0, C_1, C_2] \begin{bmatrix} \lambda I_\ell & , & F_1 & , & F_2 \\ 0 & , & \lambda I_{n-\ell} - A_{11} & , & -A_{12} \\ -gB_2K_1 & , & -A_{21}+gB_2K_oF_1 & , & \lambda I_\ell - A_{22} + gB_2K_oF_2 \end{bmatrix}^{-1} \begin{bmatrix} I_\ell \\ 0 \\ gB_2K_o \end{bmatrix} \tag{4.13}$$

and the high-gain tracking characteristics of this system can accordingly be elucidated by invoking the results of Porter and Shenton (1975) from the singular perturbation analysis of transfer function matrices.

These results yield the asymptotic form of $G(\lambda)$ as the gain parameter $g \to \infty$ and thus greatly facilitate the determination of controller and transducer matrices K_0, K_1, and M such that the continuous-time tracking behaviour of the closed-loop system becomes increasingly non-interacting as g is increased. The frequency-response and step-response characteristics of a continuous-time flight-control system for the longitudinal dynamics of an aircraft (Kouvaritakis, Murray, and MacFarlane 1979) are presented in order to illustrate these general results.

4.2 Analysis

It is evident from equation (4.13) that, by regarding $\varepsilon = 1/g$ as the perturbation parameter, the asymptotic form of the transfer function matrix $G(\lambda)$ of the continuous-time tracking system as $g \to \infty$ can be determined by invoking the results of Porter and Shenton (1975) from the singular perturbation analysis of transfer function matrices. Indeed, these results indicate that as $g \to \infty$ the transfer function matrix $G(\lambda)$ assumes the asymptotic form

$$\Gamma(\lambda) = \tilde{\Gamma}(\lambda) + \hat{\Gamma}(\lambda) \tag{4.14}$$

where

$$\tilde{\Gamma}(\lambda) = C_o(\lambda I_n - A_o)^{-1}B_o \quad , \tag{4.15}$$

$$\hat{\Gamma}(\lambda) = C_2(\lambda I_\ell - gA_4)^{-1}gB_2K_o \quad , \tag{4.16}$$

$$A_o = \begin{bmatrix} -K_o^{-1}K_1 & , & 0 \\ A_{12}F_2^{-1}K_o^{-1}K_1 & , & A_{11}-A_{12}F_2^{-1}F_1 \end{bmatrix} \quad , \tag{4.17}$$

$$B_o = \begin{bmatrix} 0 \\ A_{12}F_2^{-1} \end{bmatrix} \quad , \tag{4.18}$$

$$C_o = [C_2F_2^{-1}K_o^{-1}K_1 \; , \; C_1-C_2F_2^{-1}F_1] \quad , \tag{4.19}$$

and

$$A_4 = -B_2K_oF_2 \quad . \tag{4.20}$$

It follows from equations (4.14), (4.15), and (4.17) that the 'slow' modes Z_s of the tracking system correspond as $g \to \infty$ to the poles $Z_1 \bigcup Z_2$ of $\tilde{\Gamma}(\lambda)$ where

$$Z_1 = \{\lambda \in C : |\lambda K_o + K_1| = 0\} \tag{4.21}$$

and

$$Z_2 = \{\lambda \in C : |\lambda I_{n-\ell} - A_{11} + A_{12}F_2^{-1}F_1| = 0\} \quad , \tag{4.22}$$

and from equations (4.14), (4.16), and (4.20) that the 'fast' modes Z_f of the tracking system correspond as $g \to \infty$ to the poles Z_3 of $\hat{\Gamma}(\lambda)$ where

$$Z_3 = \{\lambda \in C : |\lambda I_\ell + gF_2B_2K_o| = 0\} \quad . \tag{4.23}$$

Furthermore, it follows from equations (4.15), (4.17), (4.18), and(4.19) that the 'slow' transfer function matrix

$$\tilde{\Gamma}(\lambda) = (C_1 - C_2F_2^{-1}F_1)(\lambda I_{n-\ell} - A_{11} + A_{12}F_2^{-1}F_1)^{-1}A_{12}F_2^{-1} \quad (4.24)$$

and from equations (4.16) and (4.20) that the 'fast' transfer function matrix

$$\hat{\Gamma}(\lambda) = C_2F_2^{-1}(\lambda I_\ell + gF_2B_2K_o)^{-1}gF_2B_2K_o \quad . \quad (4.25)$$

Hence, in view of equations (4.24) and (4.25), it is evident from equation (4.14) that as $g \to \infty$ the transfer function matrix $G(\lambda)$ of the continuous-time tracking system assumes the asymptotic form

$$\Gamma(\lambda) = (C_1 - C_2F_2^{-1}F_1)(\lambda I_{n-\ell} - A_{11} + A_{12}F_2^{-1}F_1)^{-1}A_{12}F_2^{-1}$$

$$+ C_2F_2^{-1}(\lambda I_\ell + gF_2B_2K_o)^{-1}gF_2B_2K_o \quad (4.26)$$

in consonance with the fact that both the 'slow' modes corresponding to the poles Z_2 and the 'fast' modes corresponding to the poles Z_3 possibly remain both controllable and observable as $g \to \infty$. However, the 'slow' transfer function matrix $\tilde{\Gamma}(\lambda)$ reduces to the form expressed by equation (4.24) precisely because the 'slow' modes corresponding to the poles Z_1 definitely become asymptotically uncontrollable as $g \to \infty$ in view of the block structure of the matrices A_o and B_o in equations (4.17) and (4.18).

4.3 SYNTHESIS

It is evident from equations (4.7), (4.9), (4.11), and (4.12) that tracking will occur in the sense of equation (4.5) provided only that

$$Z_s \cup Z_f \subset C^- \tag{4.27}$$

where C^- is the open left half-plane. In view of equations(4.21), (4.22), and (4.23), the 'slow' and 'fast' modes will satisfy the tracking requirement(4.27) for sufficiently large gains if the controller and transducer matrices K_0, K_1, and M are chosen such that $Z_1 \subset C^-$, $Z_2 \subset C^-$, and $Z_3 \subset C^-$ in the case of plants for which the transducer matrix M can be simultaneously chosen so as to satisfy the measurement condition expressed by equation (4.10). Moreover, if K_0 and M are chosen so that both $\tilde{\Gamma}(\lambda)$ and $\hat{\Gamma}(\lambda)$ are diagonal transfer function matrices by requiring that

$$F_2B_2K_0 = (C_2+MA_{12})B_2K_0 = \mathrm{diag}\{\sigma_1,\sigma_2,\ldots,\sigma_\ell\} \tag{4.28}$$

where $\sigma_j \in R^+$ $(j=1,2,\ldots,\ell)$ in the case of $\hat{\Gamma}(\lambda)$, it follows from equation(4.20)that the transfer function matrix $G(\lambda)$ of the continuous-time tracking system will assume the diagonal asymptotic form $\Gamma(\lambda)$ and therefore that increasingly non-interacting tracking behaviour will occur as $g \to \infty$. Furthermore, such tracking behaviour will exhibit high accuracy in the face of plant-parameter variations provided that the steady-state conditions expressed by equation (4.9) correspond to 'kinematic' relationships which hold between the state variables as a consequence of the fundamental dynamical structure of the plant.

However, increasingly 'tight' tracking behaviour will not in general occur as $g \to \infty$ in view of the possible presence in the plant output vector of 'slow' modes corresponding to the poles Z_2 of the 'slow' transfer function matrix $\tilde{\Gamma}(\lambda)$.

But the presence of any such 'slow' modes is the inevitable consequence of the introduction of appropriate transducers which generate extra plant output measurements so as to ensure that rank $F_2B_2 = \ell$ and thus render plants for which rank $C_2B_2 < \ell$ amenable to high-gain error-actuated control. Indeed, increasingly 'tight' tracking behaviour is achievable as $g \to \infty$ only in the case of plants for which rank $C_2B_2 = \ell$ (Porter and Bradshaw 1979) and which are accordingly amenable to high-gain error-actuated control without the necessity for the generation of extra plant output measurements.

4.4 Illustrative Example

These general results can be conveniently illustrated by designing a high-gain error-actuated analogue flight controller for the longitudinal dynamics of an aircraft governed on T by the respective state and output equations (Kouvaritakis, Murray, and MacFarlane 1979)

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \\ \dot{x}_5(t) \end{bmatrix} = \begin{bmatrix} 0, & 1.132, & 0, & 0, & 1 \\ 0, & 0, & 0, & 1, & 0 \\ 0, & -0.1712, & -0.0538, & 0, & 0.0705 \\ 0, & 0, & 0.0485, & -0.8536, & -1.013 \\ 0, & 0, & -0.2909, & 1.0532, & -0.6959 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & , 0 , & 0 \\ 0 & , 0 , & 0 \\ -0.0012 & , 1 , & 0 \\ 0.4419 & , 0 , & -1.6646 \\ 0.1575 & , 0 , & -0.0732 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} \qquad (4.29)$$

and

$$\begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = \begin{bmatrix} 1, & 0, & 0, & 0, & 0 \\ 0, & 0, & 1, & 0, & 0 \\ 0, & 1, & 0, & 0, & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix} \qquad (4.30)$$

from which it can be readily verified that the first Markov parameter

$$C_2B_2 = \begin{bmatrix} 0 & , 0 & , 0 \\ -0.0012 & , 1 & , 0 \\ 0 & , 0 & , 0 \end{bmatrix} \qquad (4.31)$$

is rank defective. In case $\{\sigma_1, \sigma_2, \sigma_3\} = \{1,1,1\}$, $K_1 = K_0$, and

$$M = \begin{bmatrix} 0.25, & 0 \\ 0, & 0 \\ 0 & , 0.25 \end{bmatrix} , \qquad (4.32)$$

it follows from equations (4.3), (4.4), and (4.28) that the corresponding transducers and high-gain error-actuated analogue controller are governed on T by the respective measurement and control-law equations

$$\begin{bmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \end{bmatrix} = \begin{bmatrix} 1, & 0.283, & 0, & 0 & , 0.25 \\ 0, & 0 & , 1, & 0 & , 0 \\ 0, & 1 & , 0, & 0.25, & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix} \qquad (4.33)$$

and

$$
\begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} = g\left\{ \begin{bmatrix} -28.97 & , 0 , & -1.274 \\ -0.0348 & , 1 , & -0.0015 \\ -7.690 & , 0 , & -2.740 \end{bmatrix} \begin{bmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \end{bmatrix} + \begin{bmatrix} -28.97 & , 0 , & -1.274 \\ -0.0348 & , 1 , & -0.0015 \\ -7.690 & , 0 , & -2.740 \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \end{bmatrix} \right\} \qquad (4.34)
$$

and it is evident from equations (4.21), (4.22), and (4.23) that $Z_1 = \{-1,-1,-1\}$, $Z_2 = \{-4,-4\}$, and $Z_3 = \{-g,-g,-g\}$. It is also evident from equation (4.26) that the asymptotic transfer function matrix assumes the diagonal form

$$
\Gamma(\lambda) = \begin{bmatrix} \frac{4}{\lambda+4} & , & 0 & , & 0 \\ 0 & , & \frac{g}{\lambda+g} & , & 0 \\ 0 & , & 0 & , & \frac{4}{\lambda+4} \end{bmatrix} \qquad (4.35)
$$

and therefore that the closed-loop continuous-time flight-control system for the longitudinal dynamics of the aircraft will exhibit increasingly non-interacting tracking behaviour as $g \to \infty$ when the control input vector $[u_1(t)\ ,\ u_2(t)\ ,\ u_3(t)]^T$ is generated by the high-gain analogue controller governed on T by equation (4.34). However, it is apparent from equation (4.35) that increasingly 'tight' tracking behaviour will be achieved only in the case of $y_2(t)$ in view of the presence in $y_1(t)$ and $y_3(t)$ of the 'slow' modes corresponding to the poles Z_2.

The actual frequency-response loci $G(i\omega)$ for $\omega \in (-\infty, +\infty)$ are shown in Figs 4.1, 4.2, and 4.3 when $g = 10, 20$, and 50, respectively, and it is clear that the actual frequency-response loci approach the asymptotic frequency-response loci $\Gamma(i\omega)$ as the gain parameter g is increased. The corresponding step-response characteristics are shown in Fig 4.4, Fig 4.5, and Fig 4.6 when $[v_1(t) \ , \ v_2(t) \ , \ v_3(t)]^T = [1 \ , \ 0 \ , \ 0]^T$, $[v_1(t) \ , \ v_2(t) \ , \ v_3(t)]^T = [0 \ , \ 1 \ , \ 0]^T$, and $[v_1(t) \ , \ v_2(t) \ , \ v_3(t)]^T = [0 \ , \ 0 \ , \ 1]^T$, respectively, and it is evident that increasingly non-interacting tracking occurs as the gain parameter g is increased but that increasingly 'tight' tracking occurs only in the case of $y_2(t)$.

4.5 CONCLUSION

Singular perturbation methods have been used to exhibit the asymptotic structure of the transfer function matrices of continuous-time tracking systems incorporating linear multivariable plants which are amenable to high-gain error-actuated analogue control only if extra plant output measurements are generated by the introduction of appropriate transducers and processed by inner-loop compensators. It has been shown that these results greatly facilitate the determination of controller and transducer matrices which ensure that the closed-loop behaviour of such continuous-time tracking systems becomes increasingly non-interacting as the gain parameter g is increased. These general results have been illustrated by the presentation of the frequency-response and step-response characteristics of a continuous-time flight-control system for the longitudinal dynamics of an aircraft.

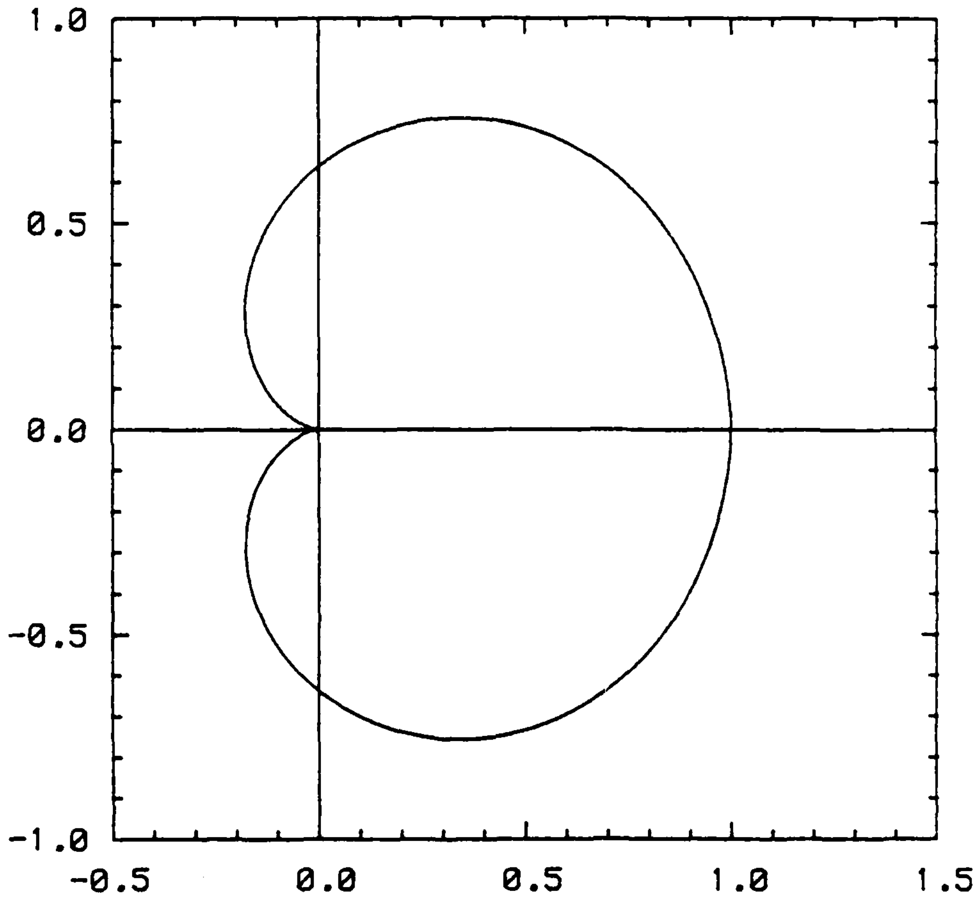


Fig 4.1(a): $G_{11}(i\omega)$; g = 10

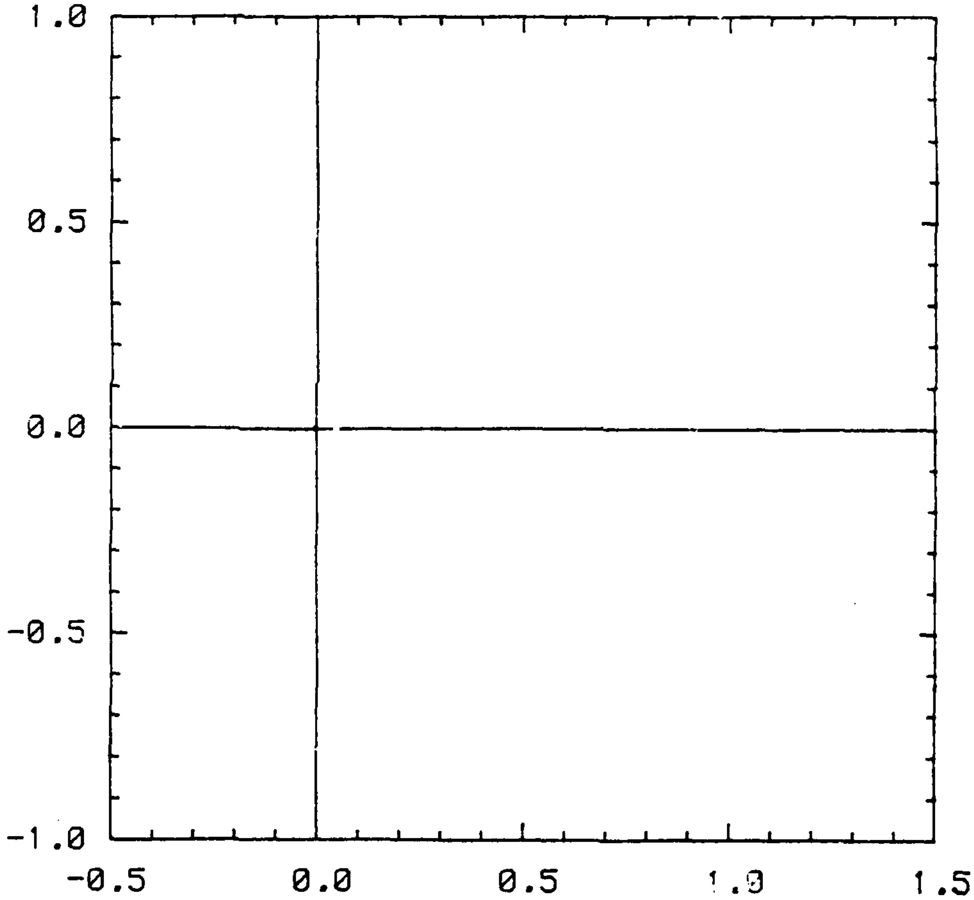


Fig 4.1(b): $G_{12}(i\omega)$; $g = 10$

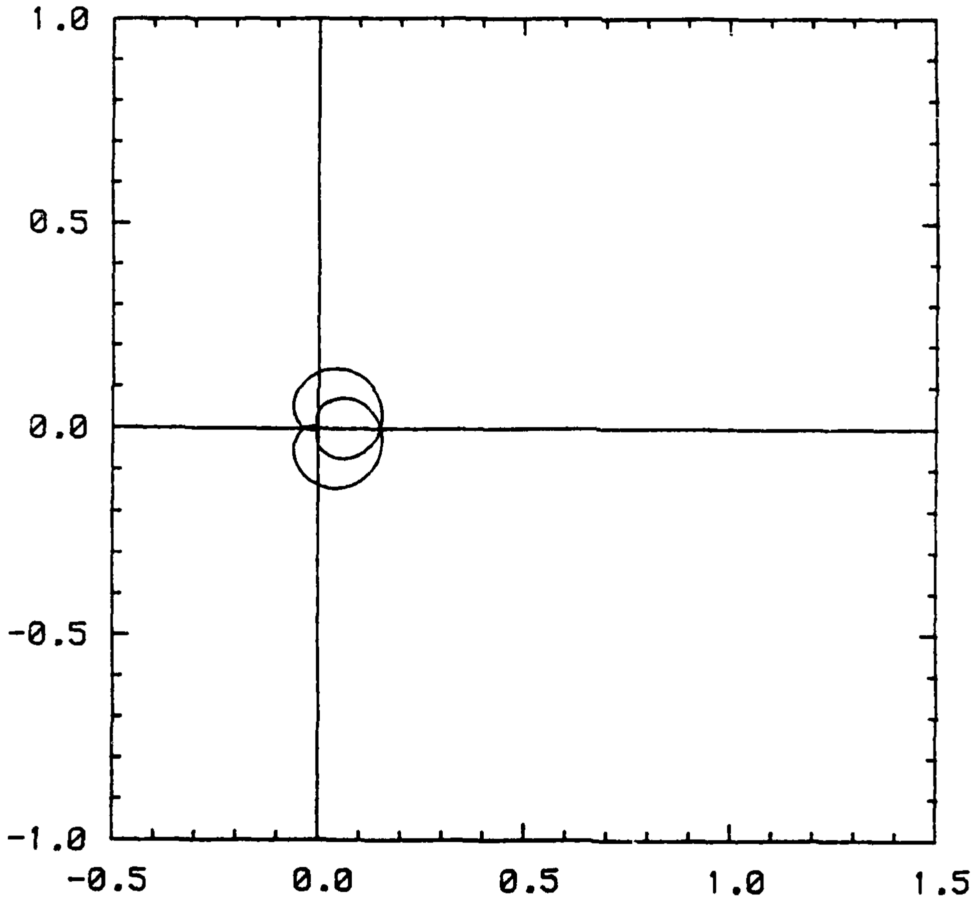


Fig 4.1(c): $G_{13}(i\omega)$: g = 10

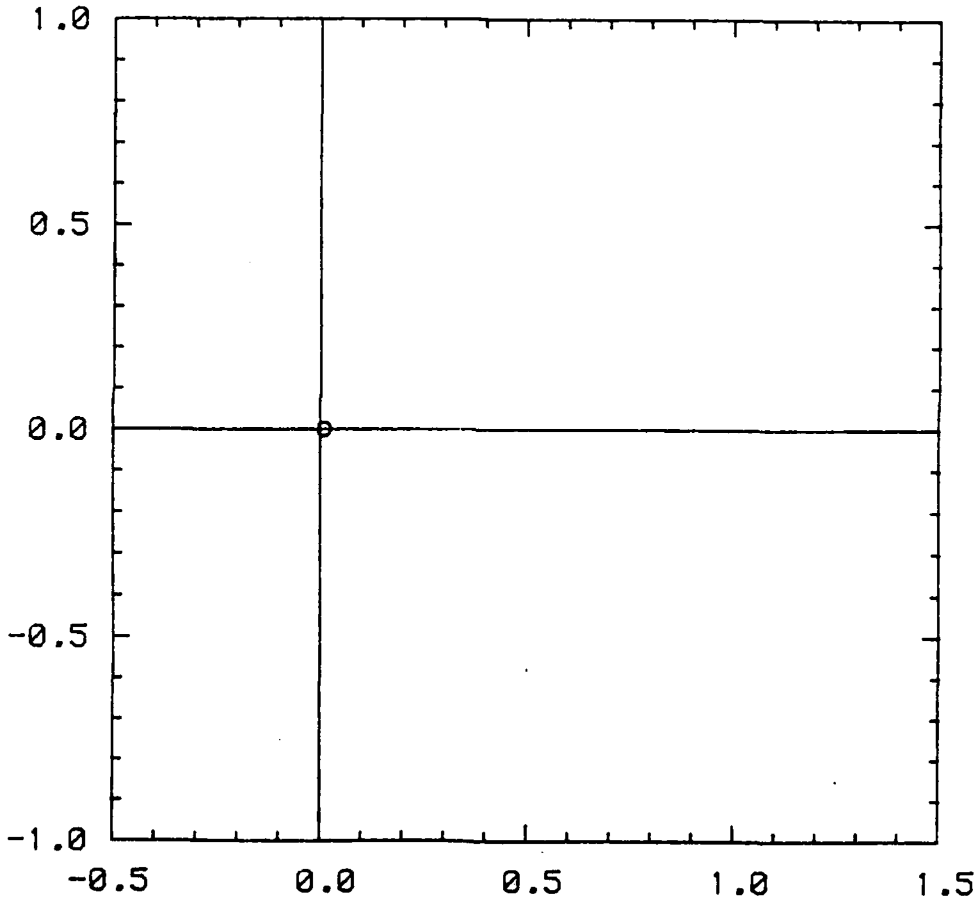


Fig 4.1(d): $G_{21}(i\omega)$; $g = 10$

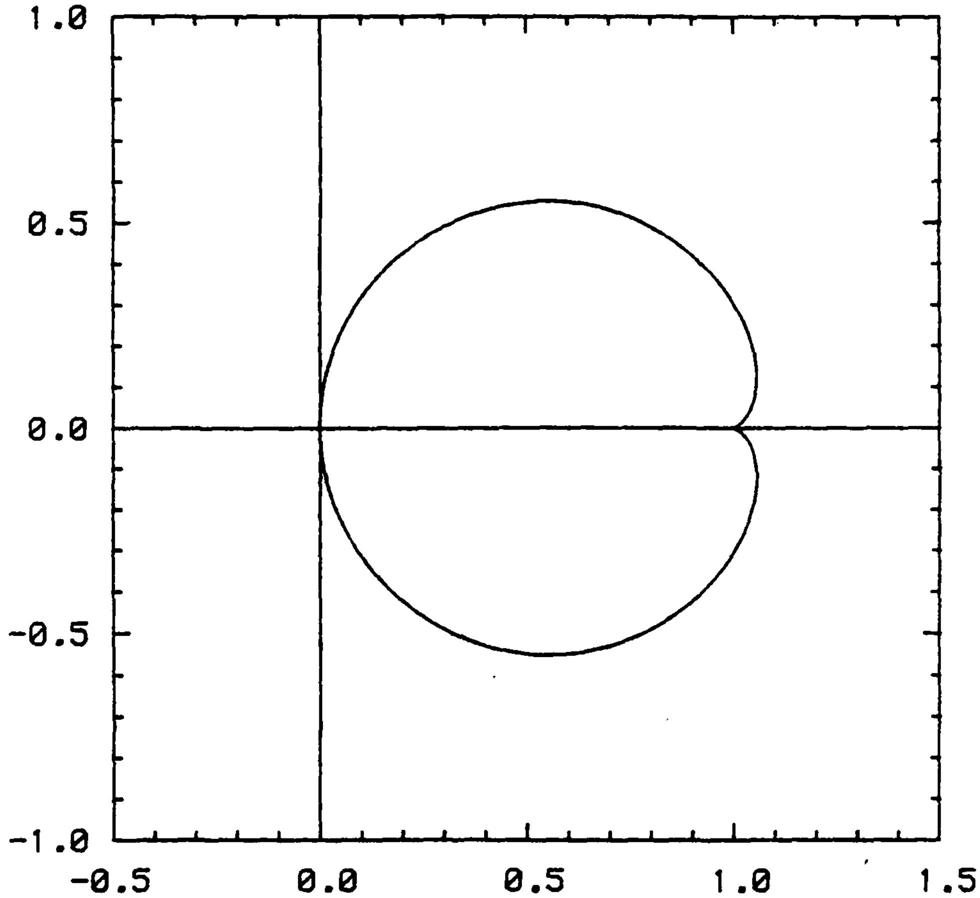


Fig 4.1(e): $G_{22}(i\omega)$; g = 10

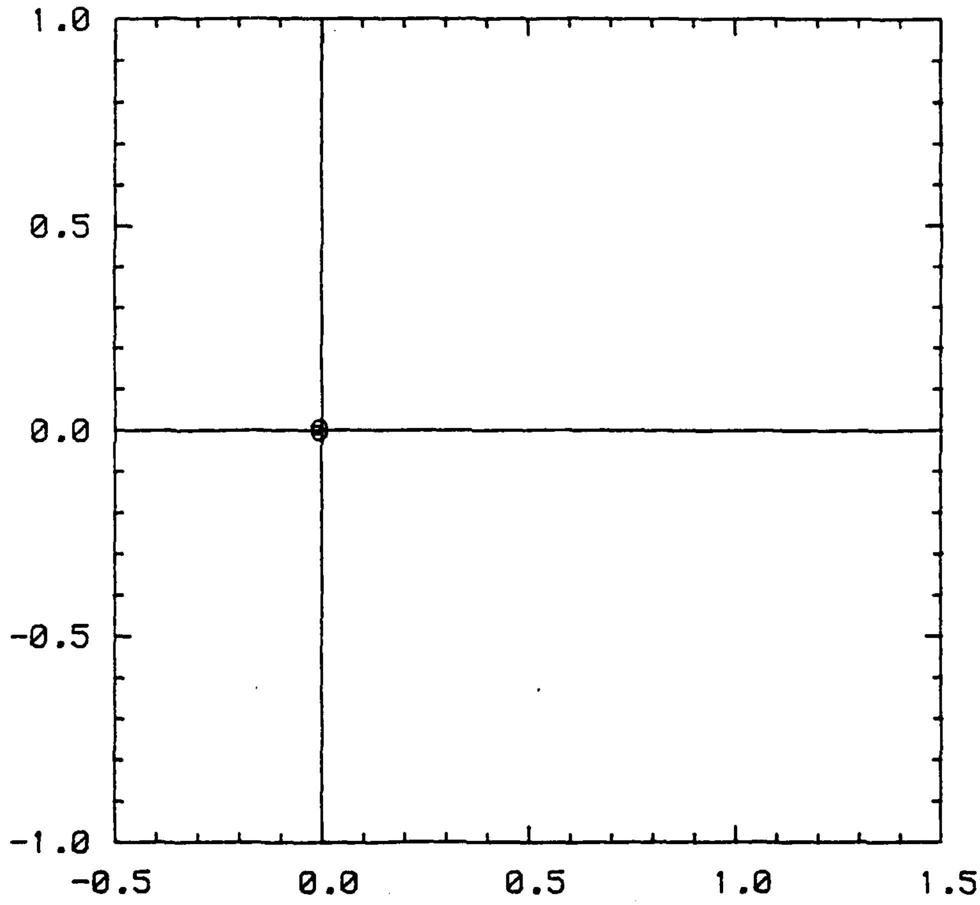


Fig 4.1(f): $G_{23}(i\omega)$; g = 10

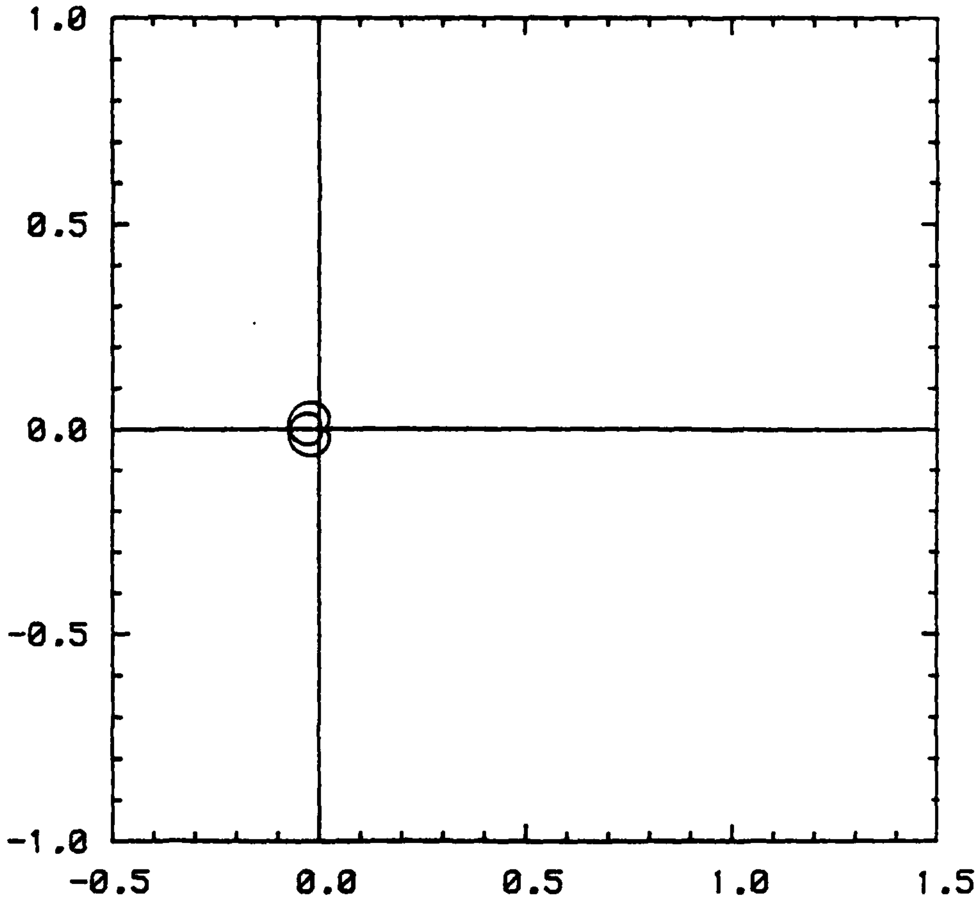


Fig 4.1(g): $G_{31}(i\omega)$; $g = 10$

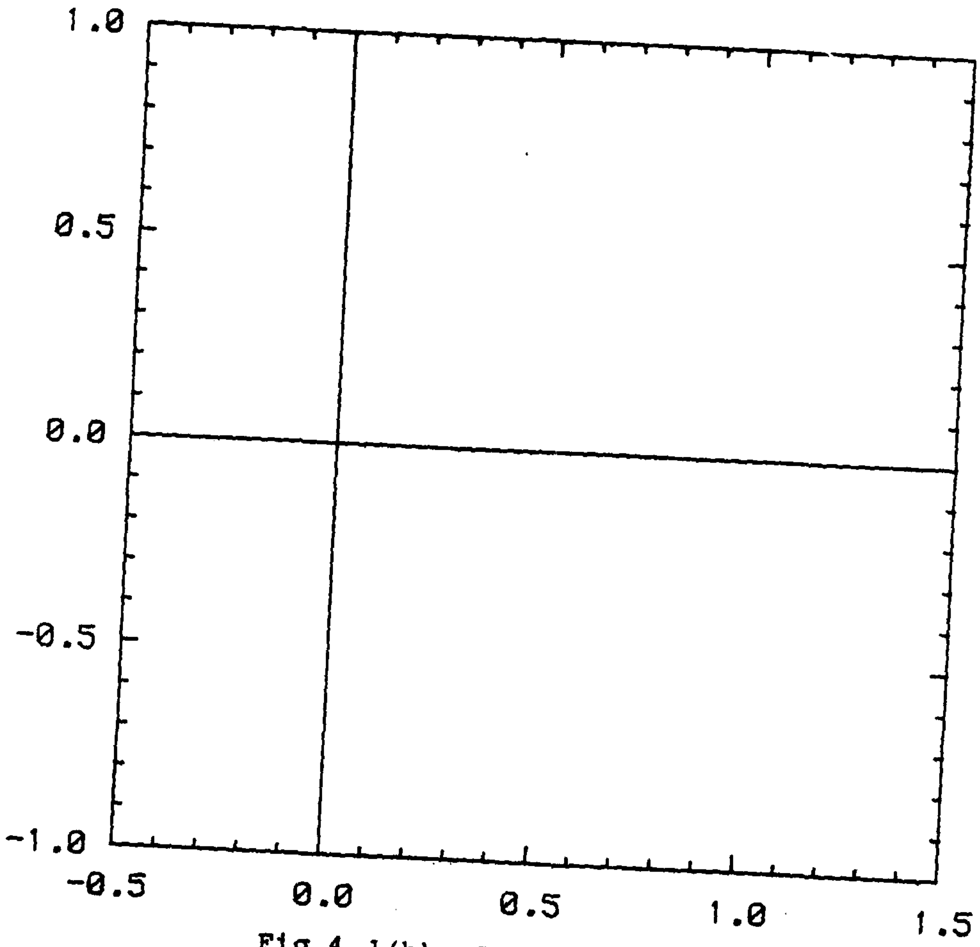


Fig 4.1(h): $G_{32}(i\omega)$; g = 10

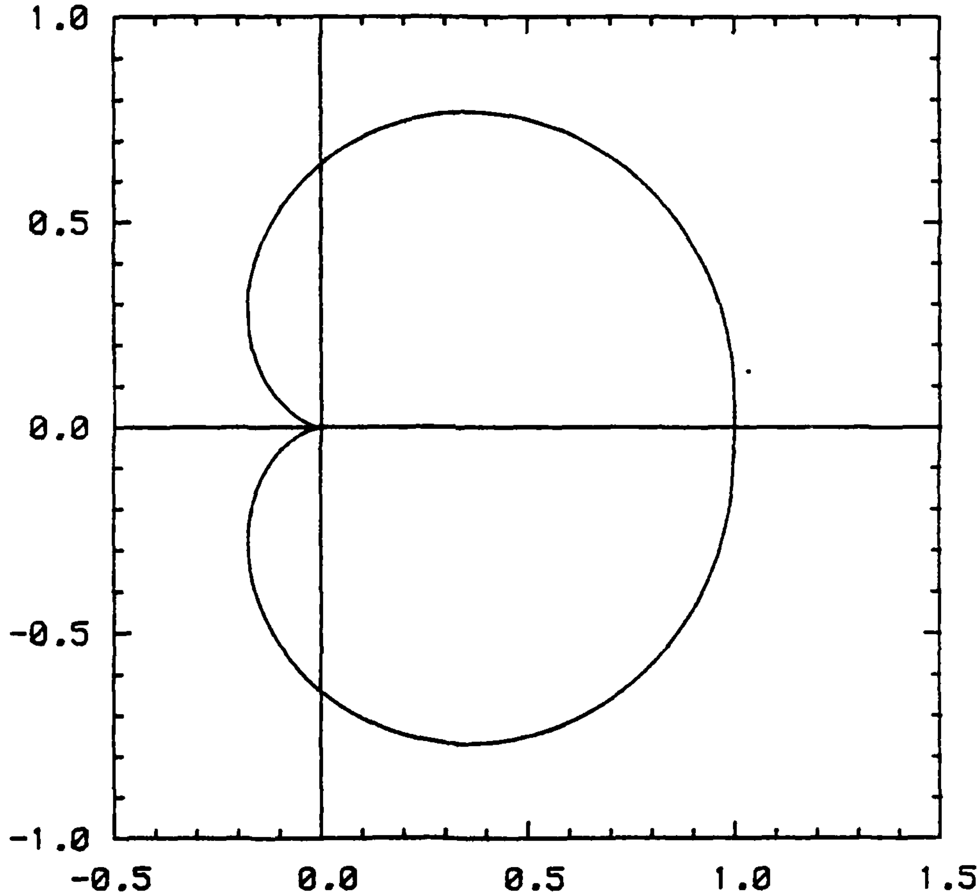


Fig 41(i): $G_{33}(i\omega)$; $g = 10$

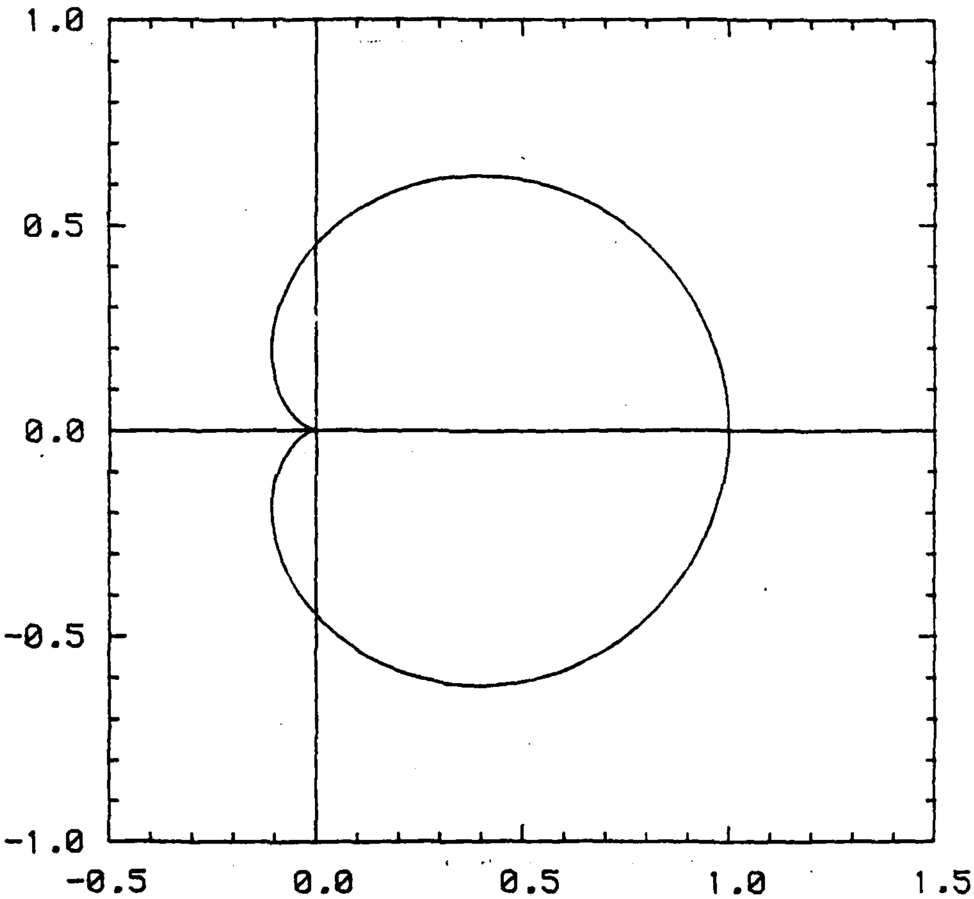


Fig 4.2(a): $G_{11}(i\omega)$; $g = 20$

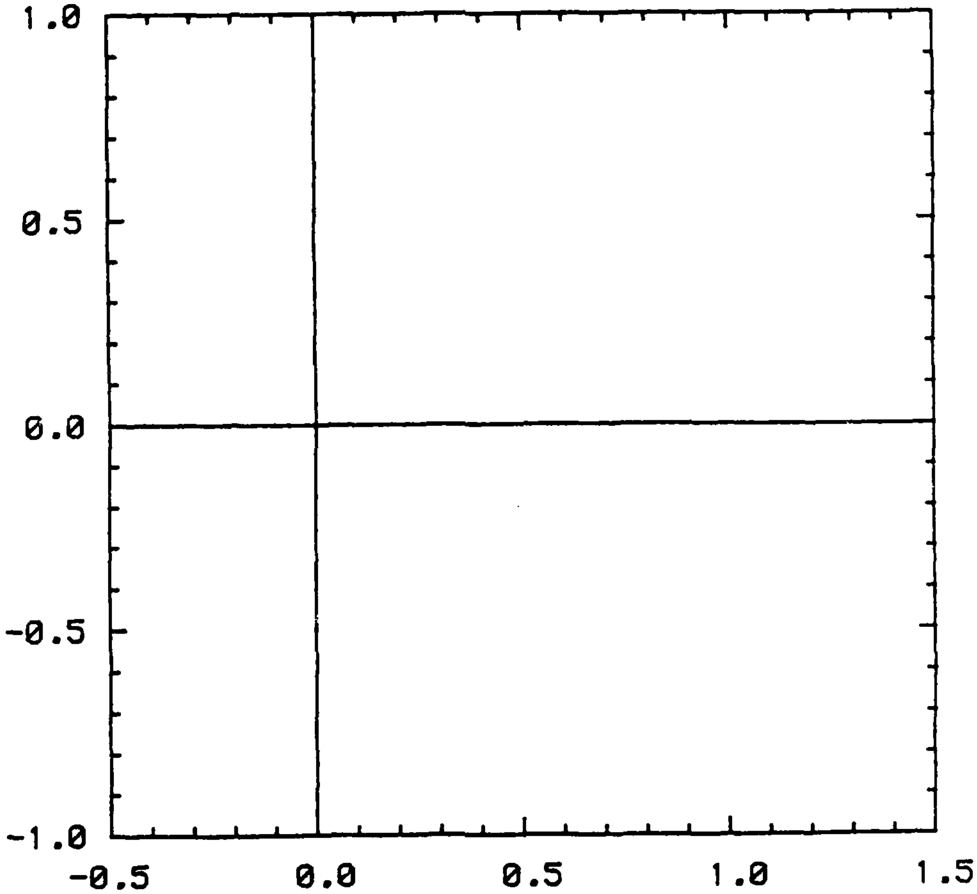


Fig 4.2(b): $G_{12}(i\omega)$; $g = 20$

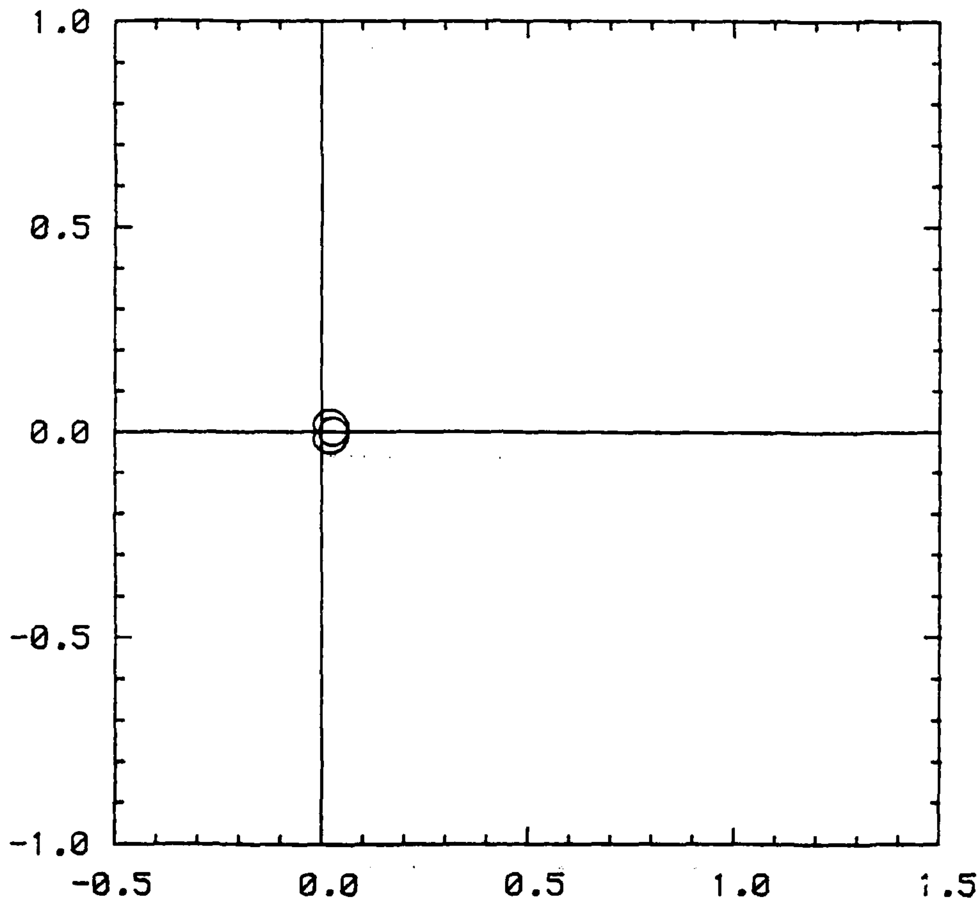


Fig 4.2(c): $G_{13}(i\omega)$; $g = 20$

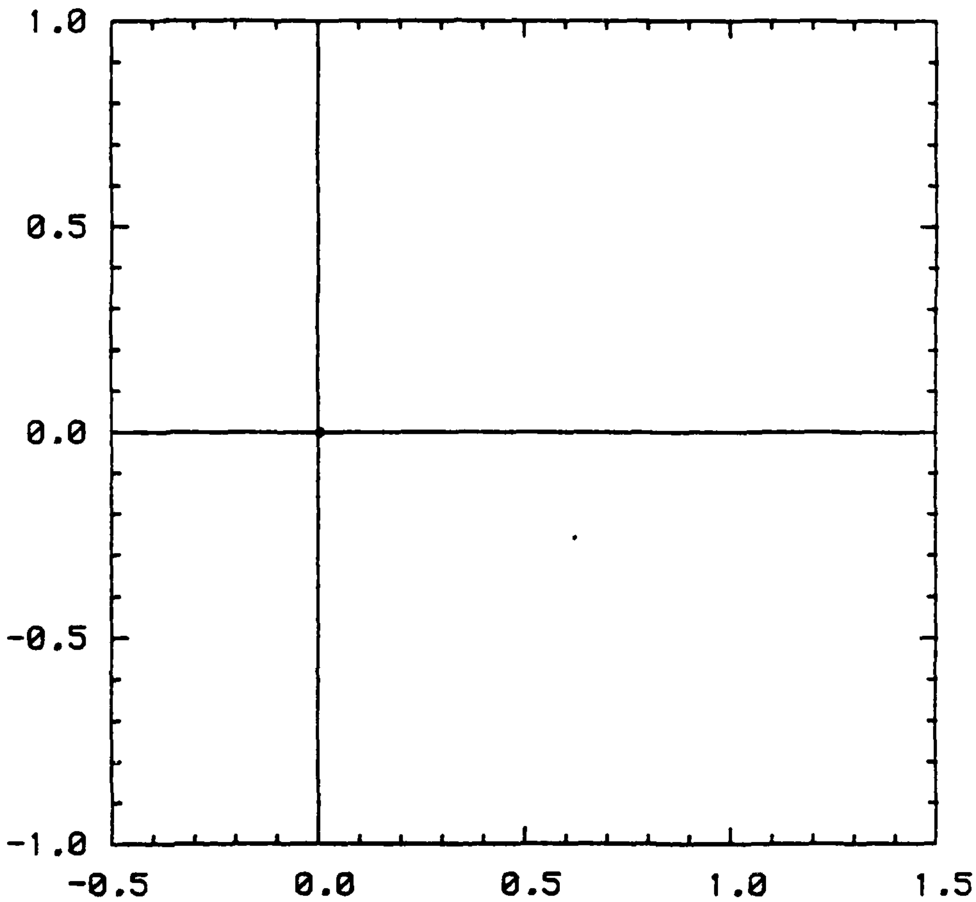


Fig 4.2(d): $G_{21}(i\omega)$; $g = 20$

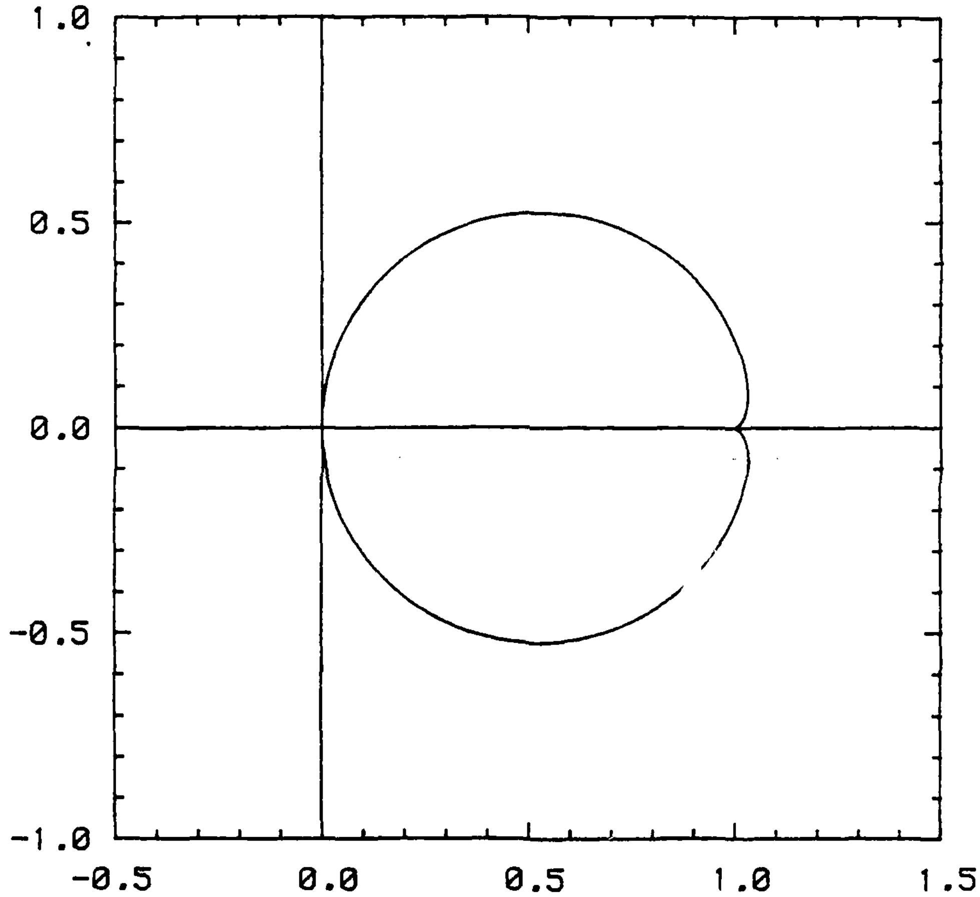


Fig 4.2(e): $G_{22}(i\omega)$; $g = 20$

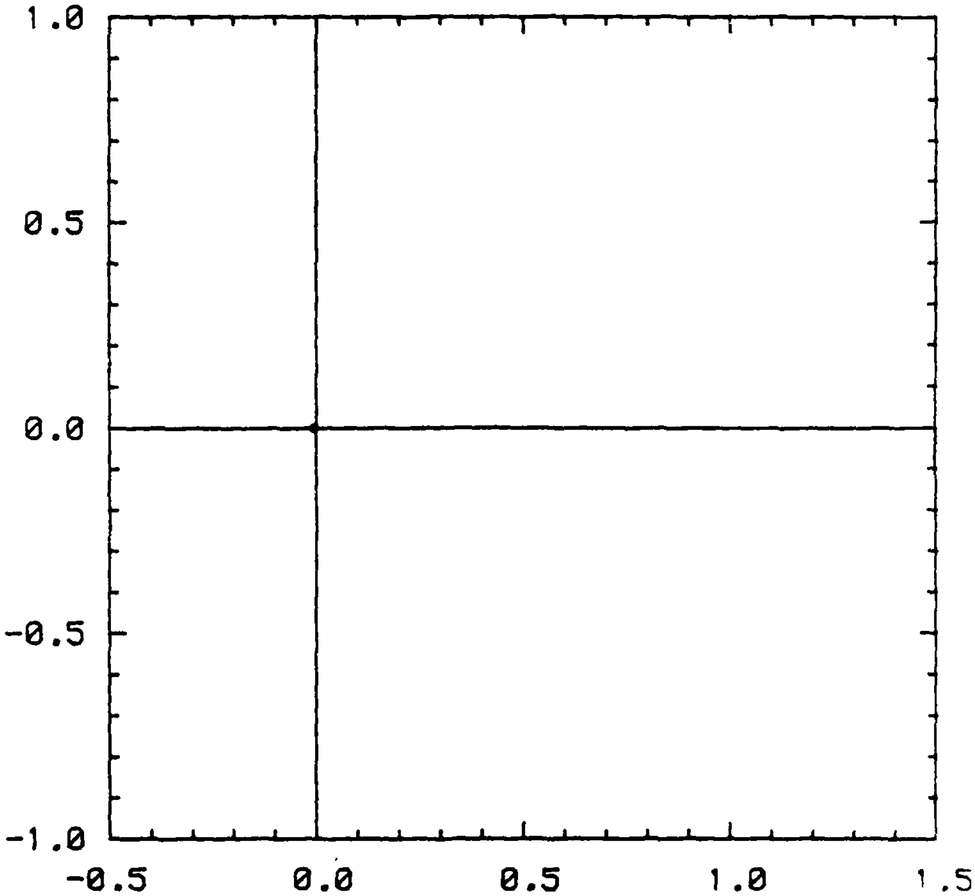


Fig 4.2(f): $G_{23}(i\omega)$; g = 20

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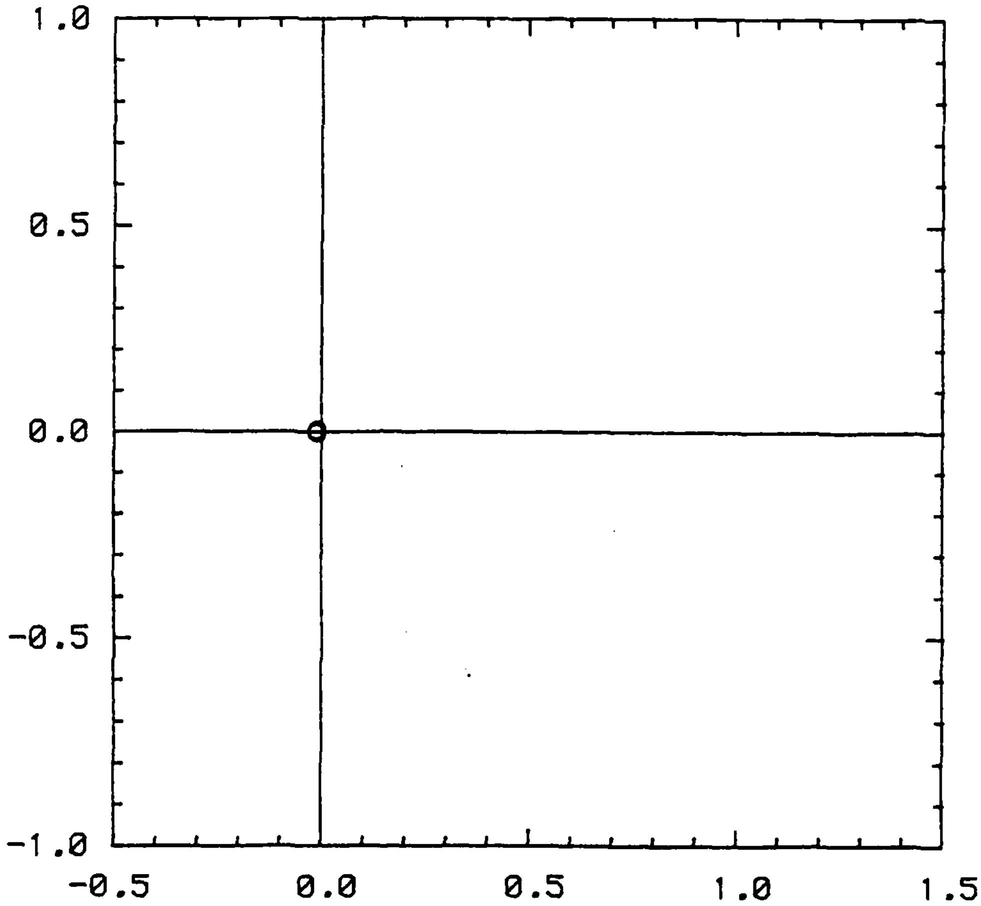

Fig 4.2(g): $G_{31}(i\omega)$; $g = 20$

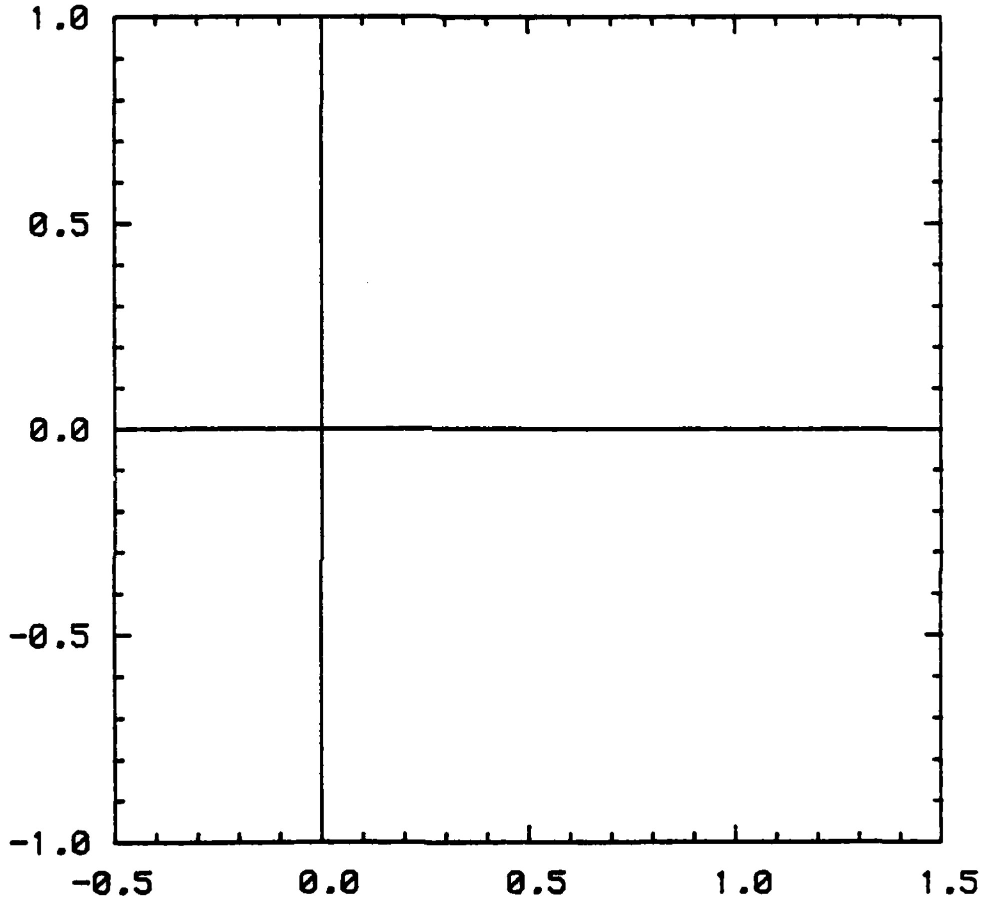

Fig 4.2(h): $G_{32}(i\omega)$; g = 20

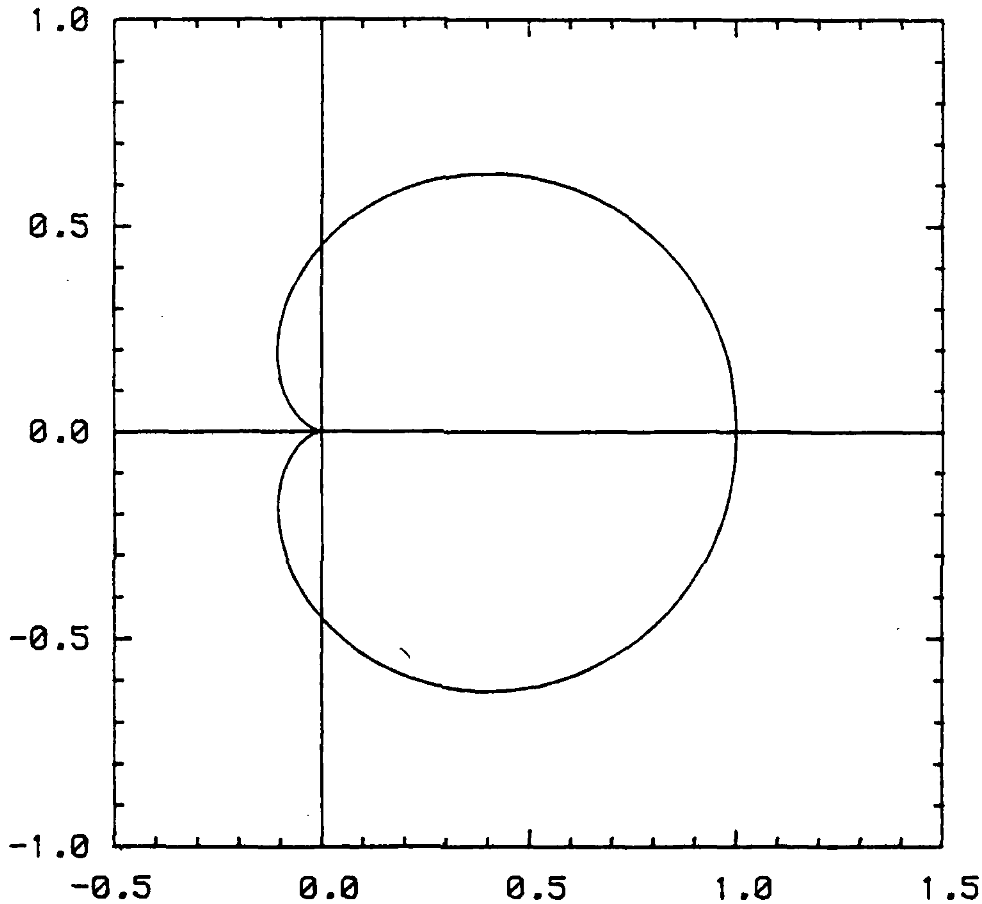

Fig 4.2(i): $G_{33}(i\omega)$; $g = 20$

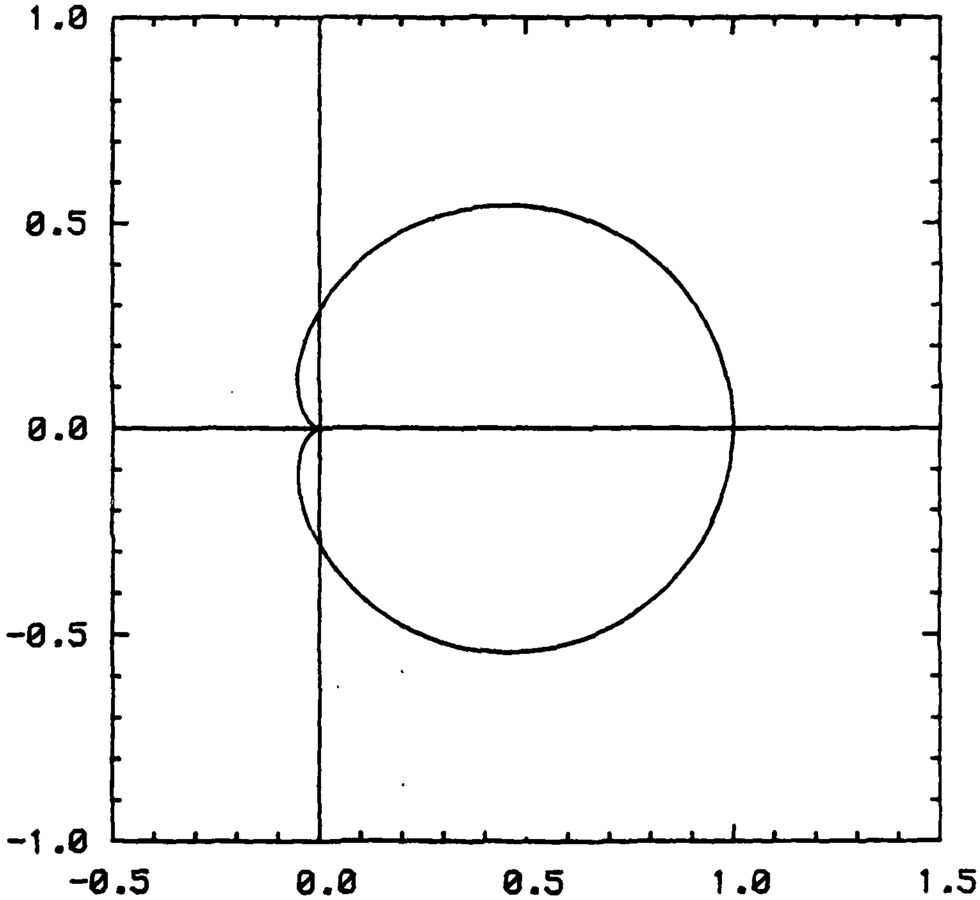

Fig 4.3(a): $G_{11}(i\omega)$; g = 50

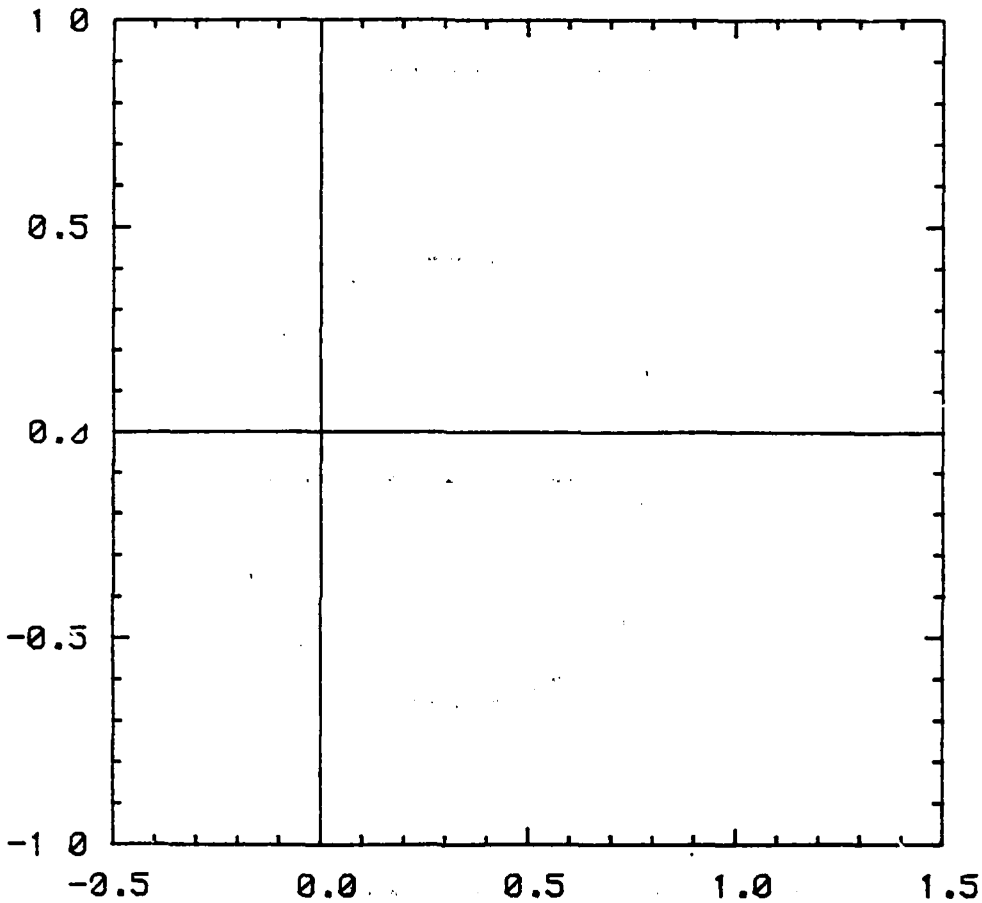

Fig 4.3(b): $G_{12}(i\omega)$; $g = 50$

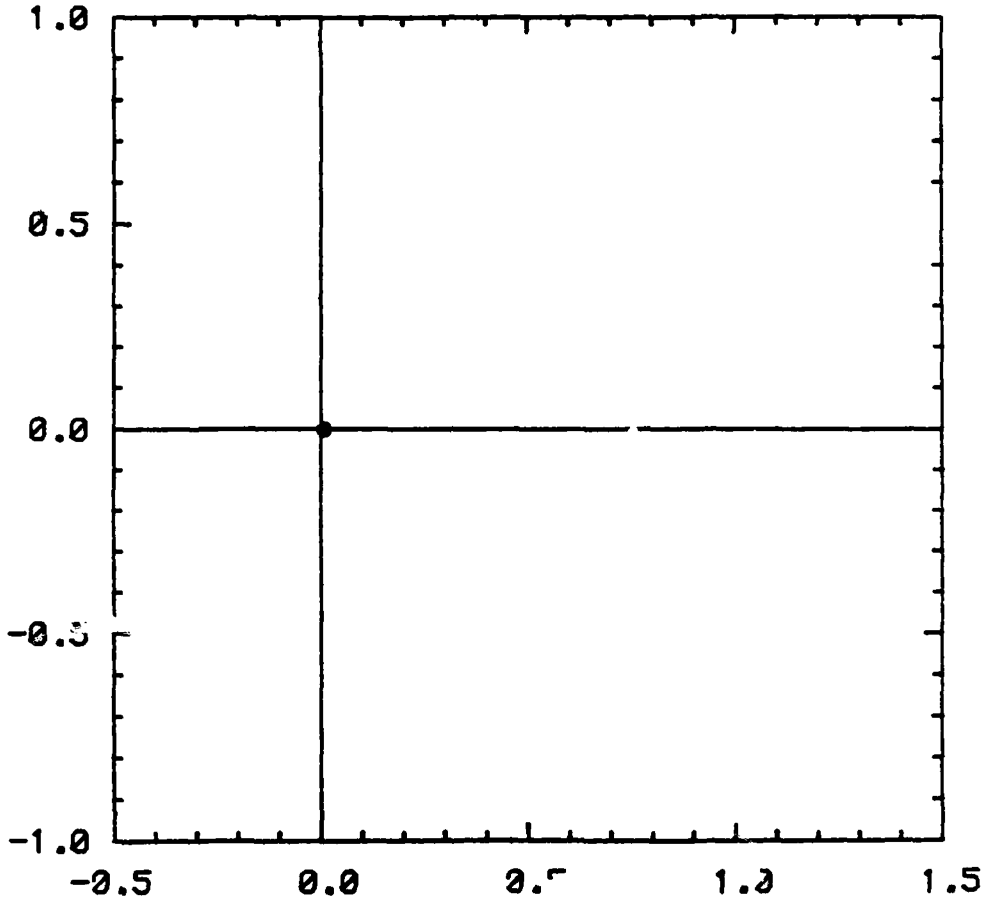

Fig 4.3(c): $G_{13}(i\omega)$; g = 50

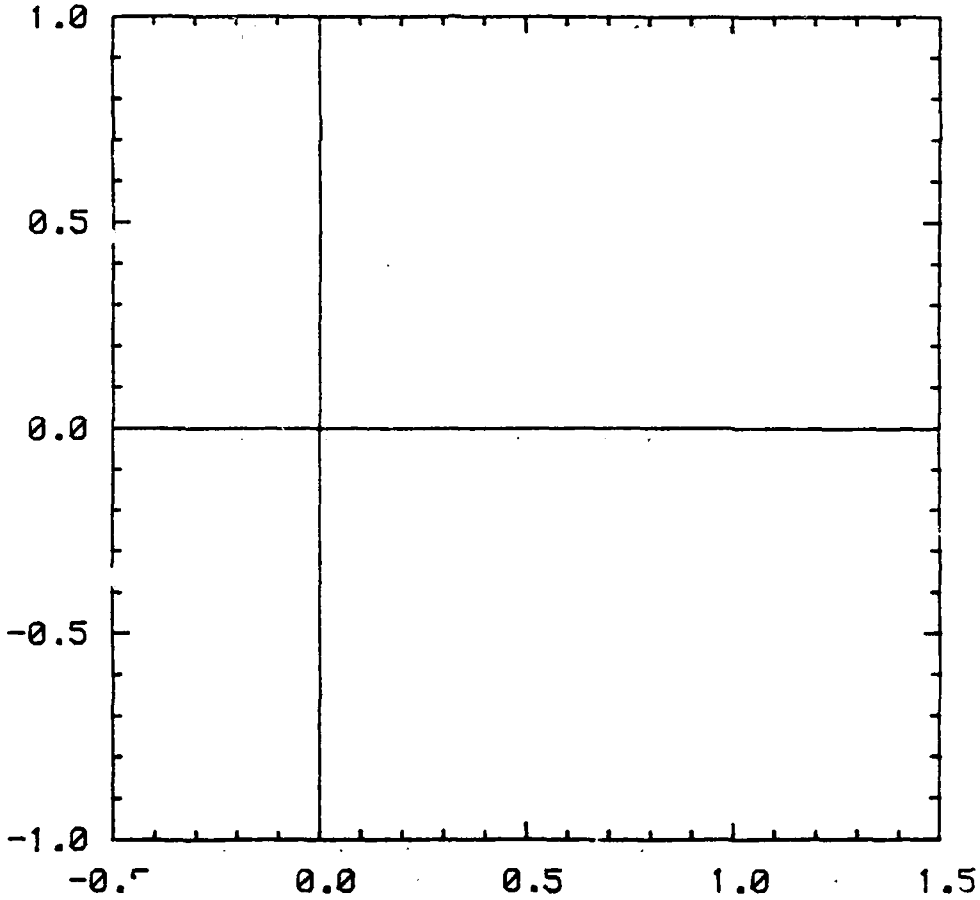

Fig 4.3(d): $G_{21}(i\omega)$; $g = 50$

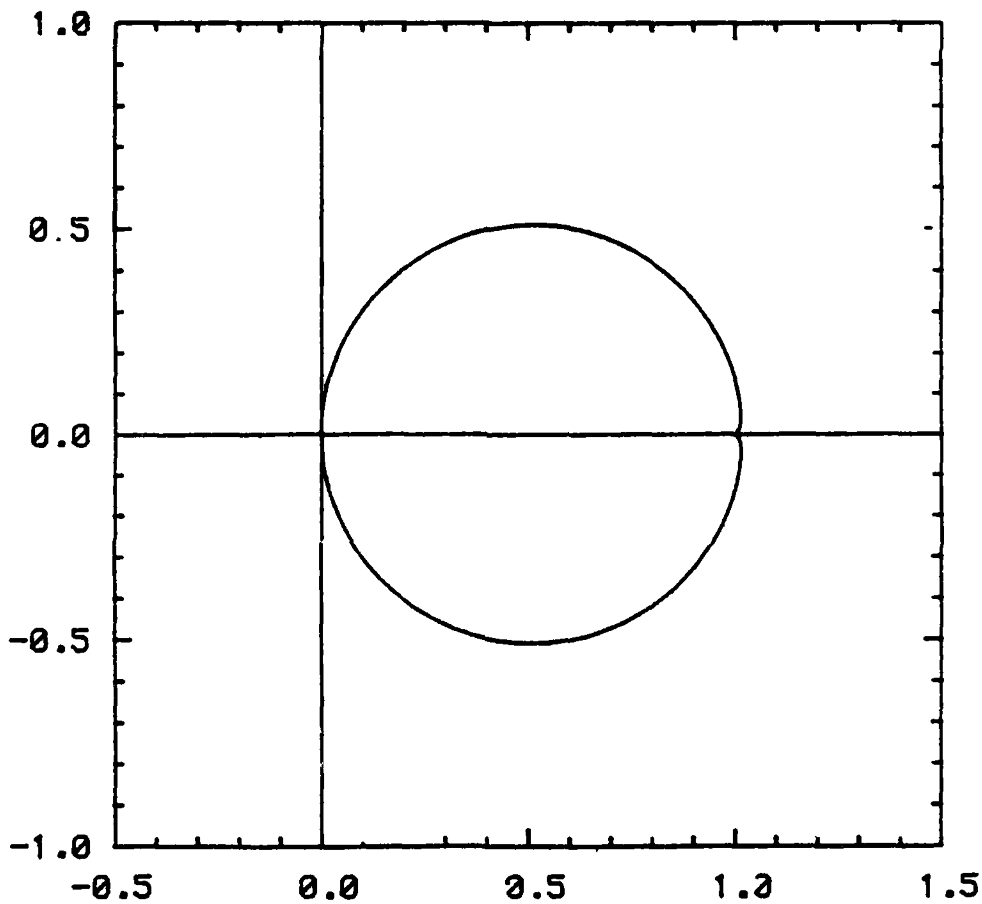

Fig 4.3(e): $G_{22}(i\omega)$; $g = 50$

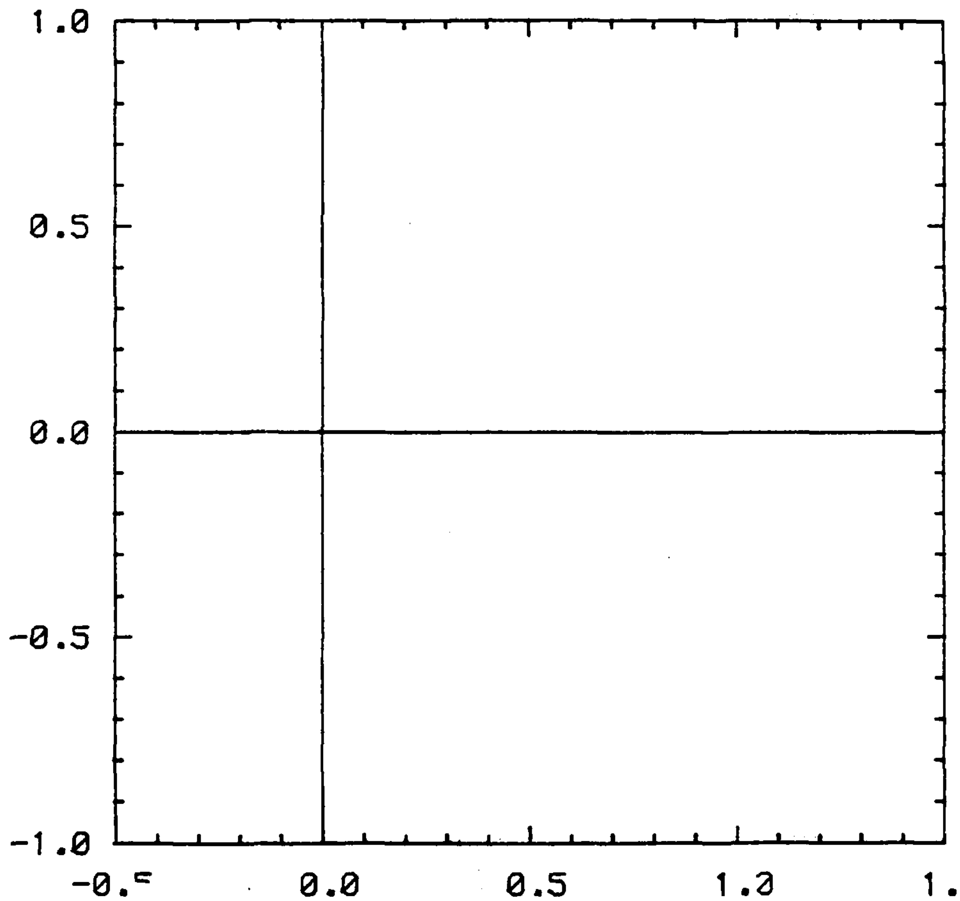

Fig 4.3(f): $G_{23}(i\omega)$; $g = 50$

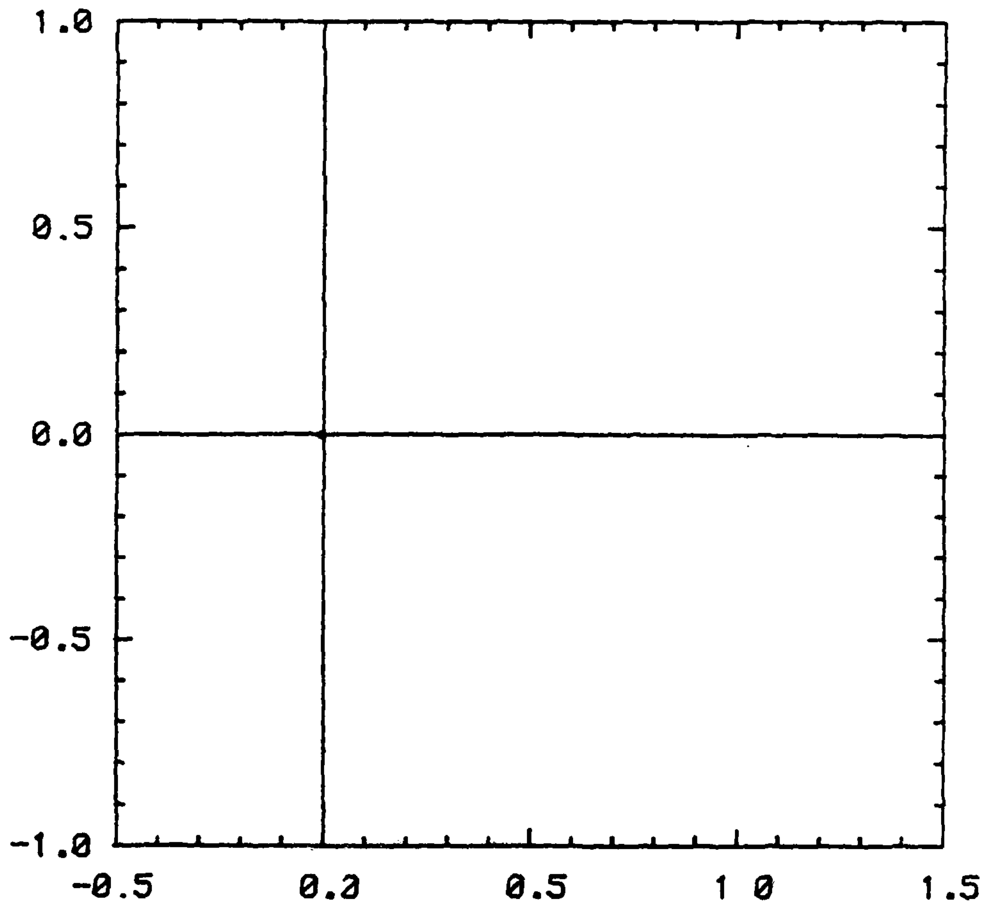

Fig 4.3(g): $G_{31}(i\omega)$; g = 50

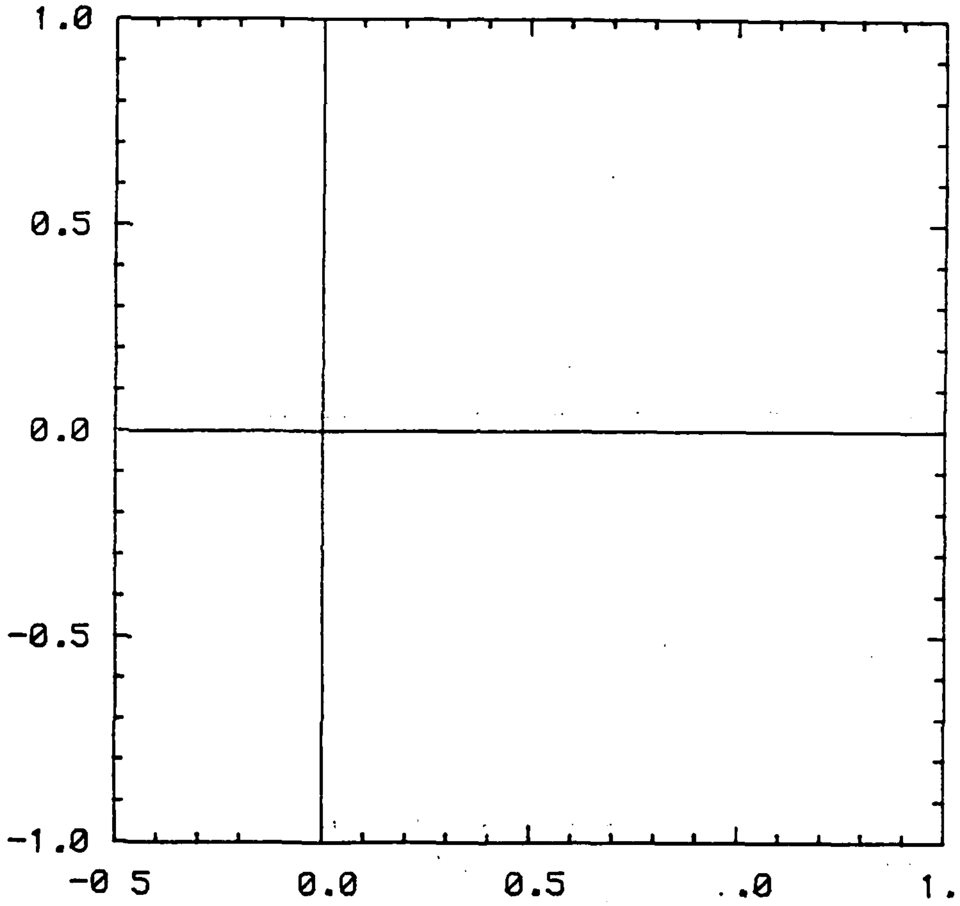

Fig 4.3(h): $G_{32}(i\omega)$; g = 50

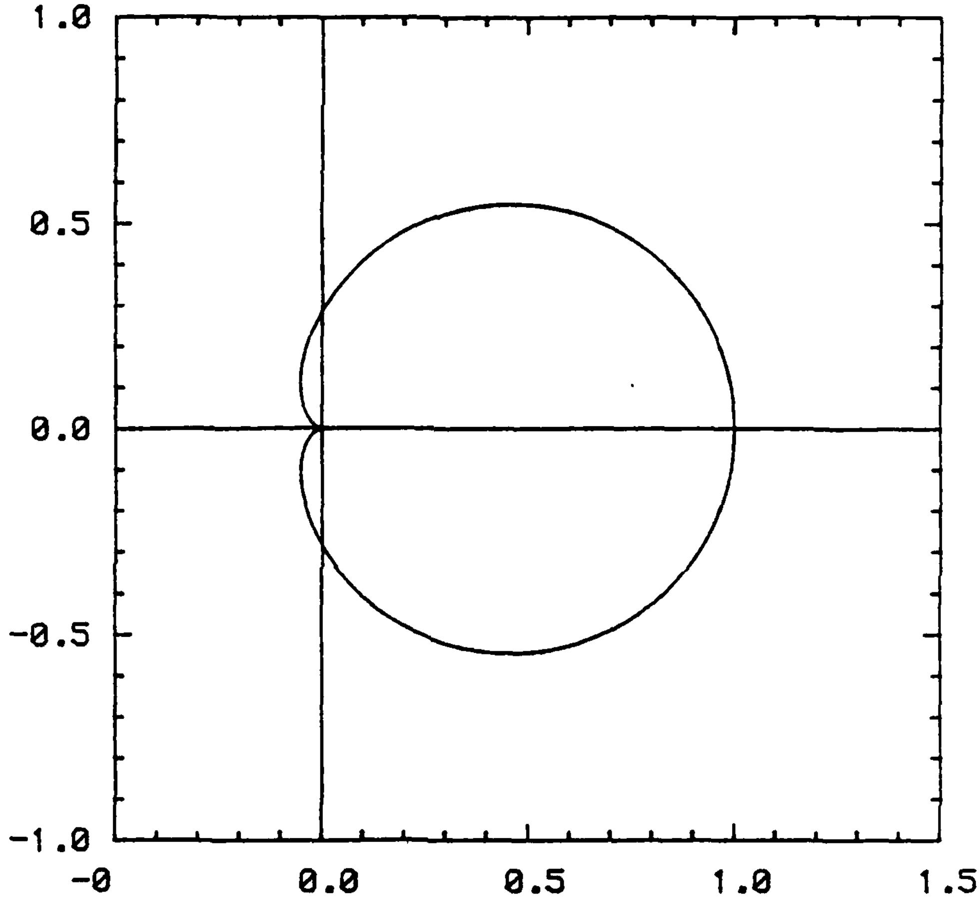

Fig 4.3(i): $G_{33}(i\omega)$; g = 50

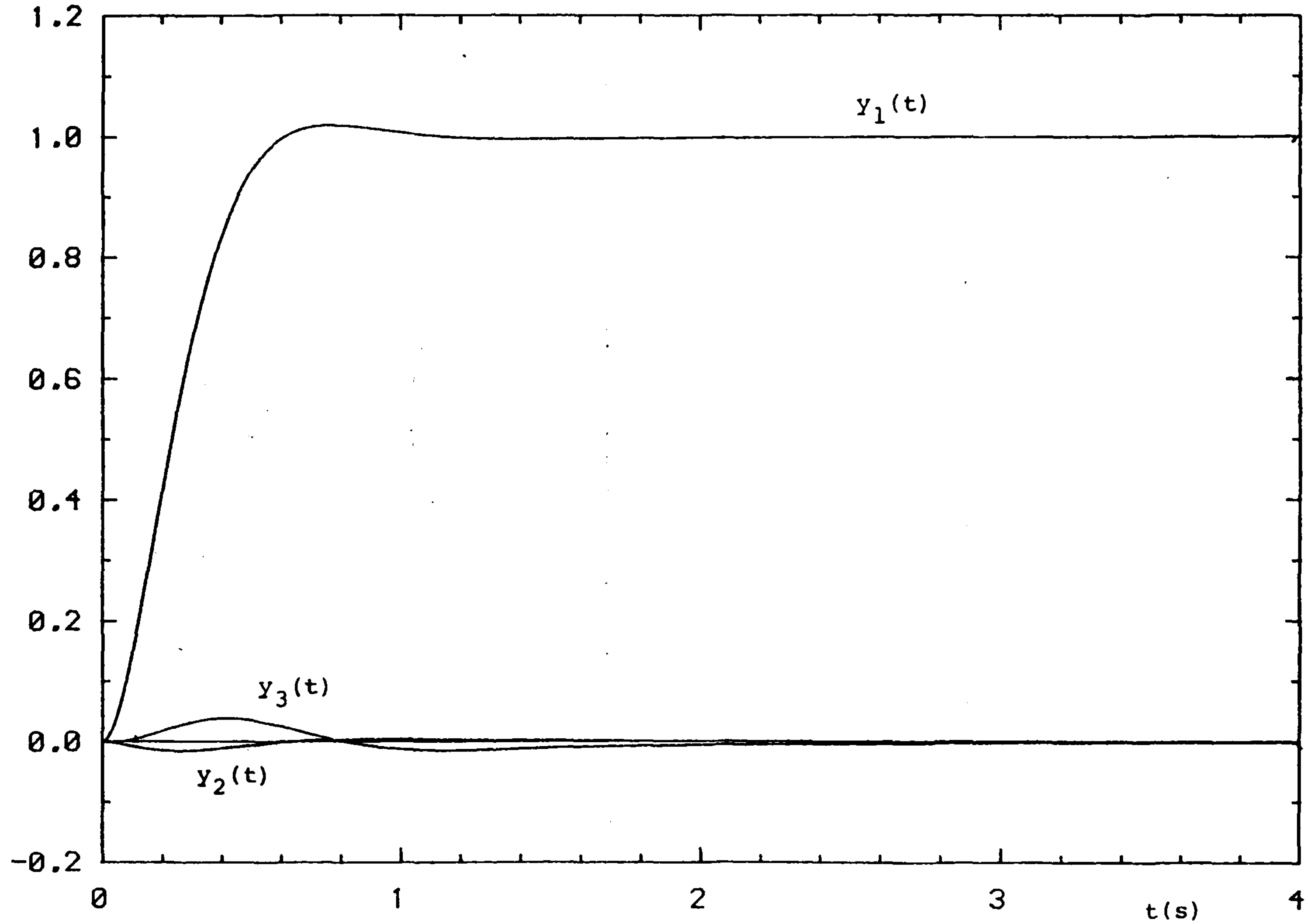

Fig 4.4(a): g = 10

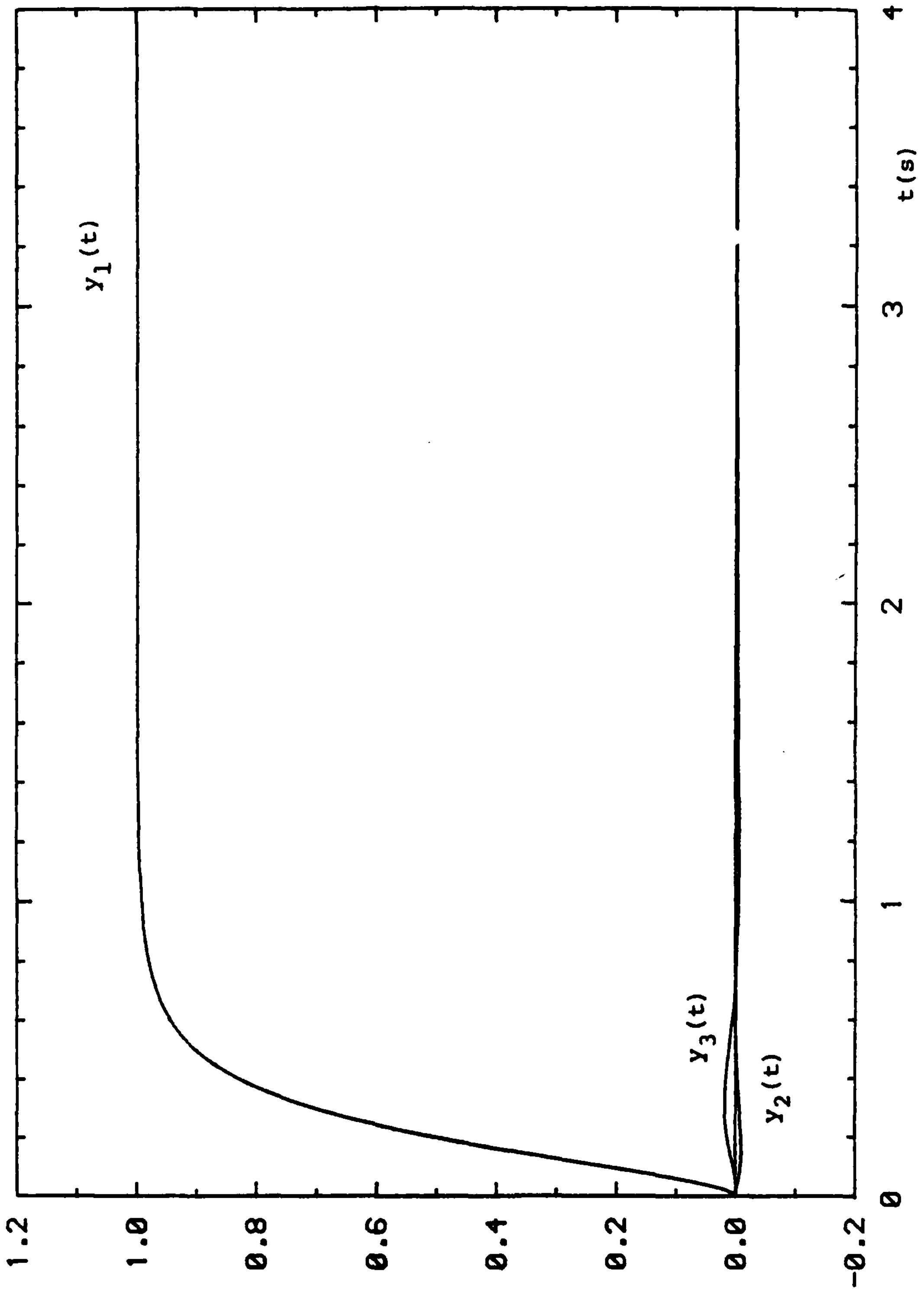

Fig 4.4(b): g = 20

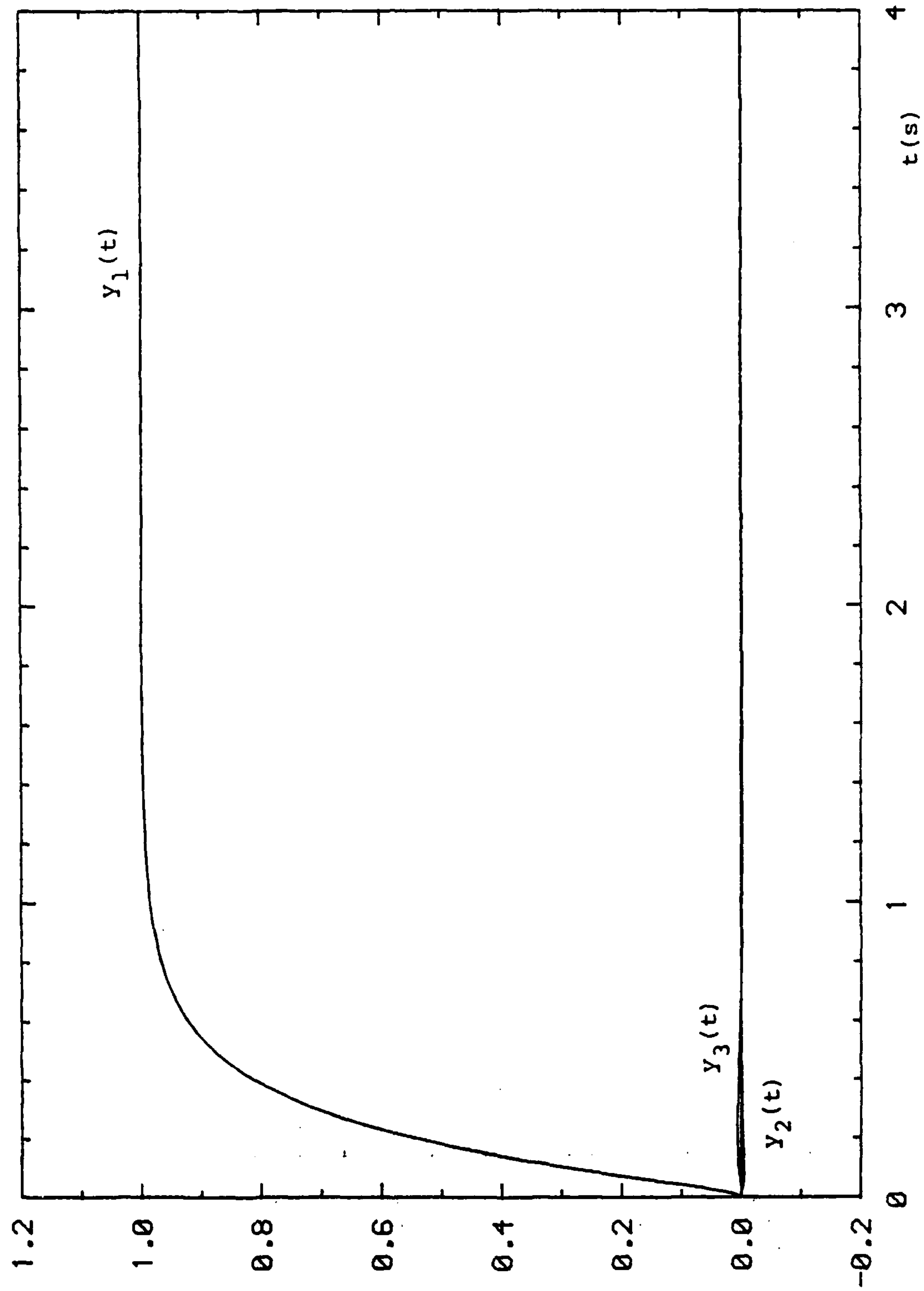

Fig 4.4(c): g = 50

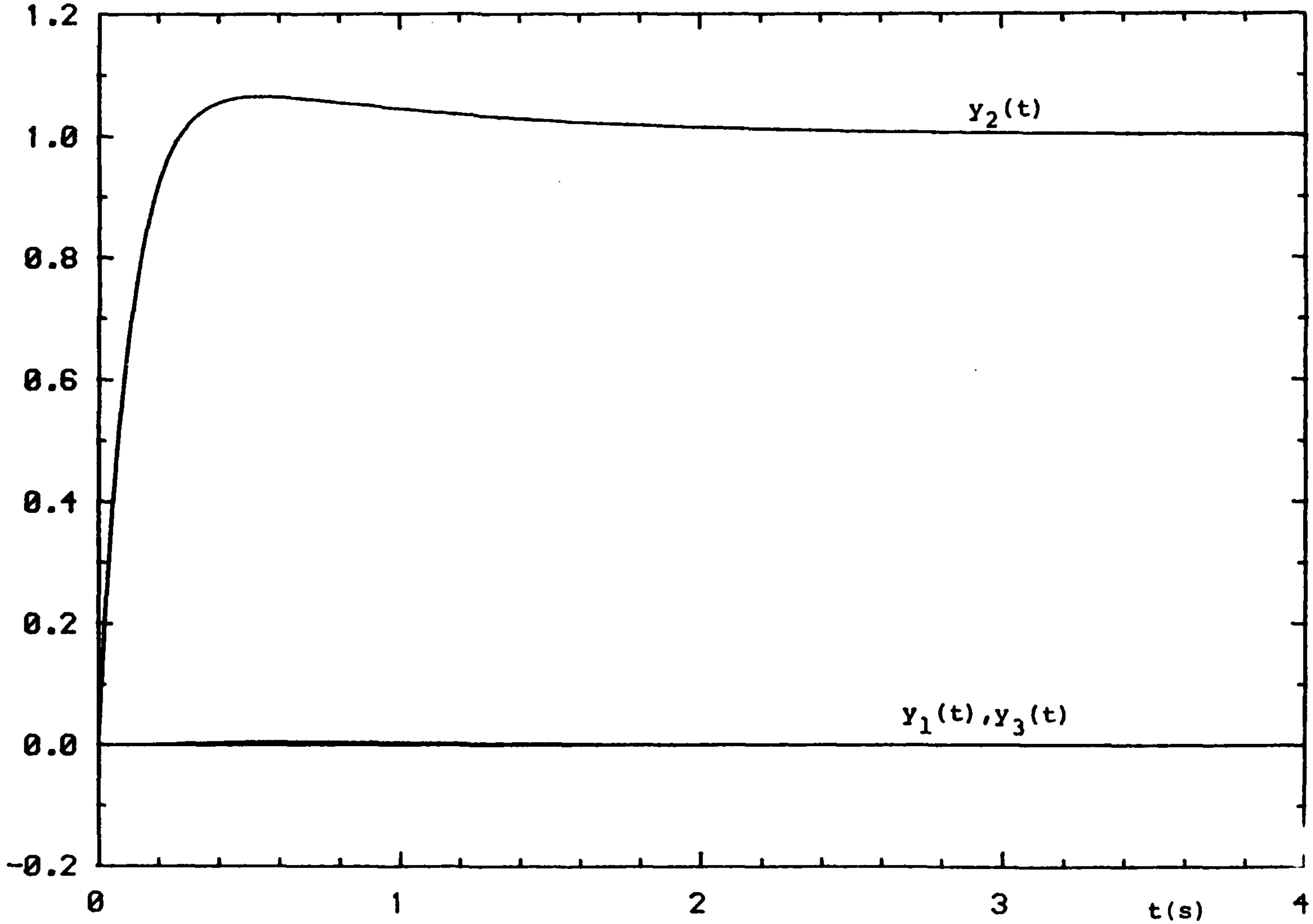

Fig 4.5(a): g = 10

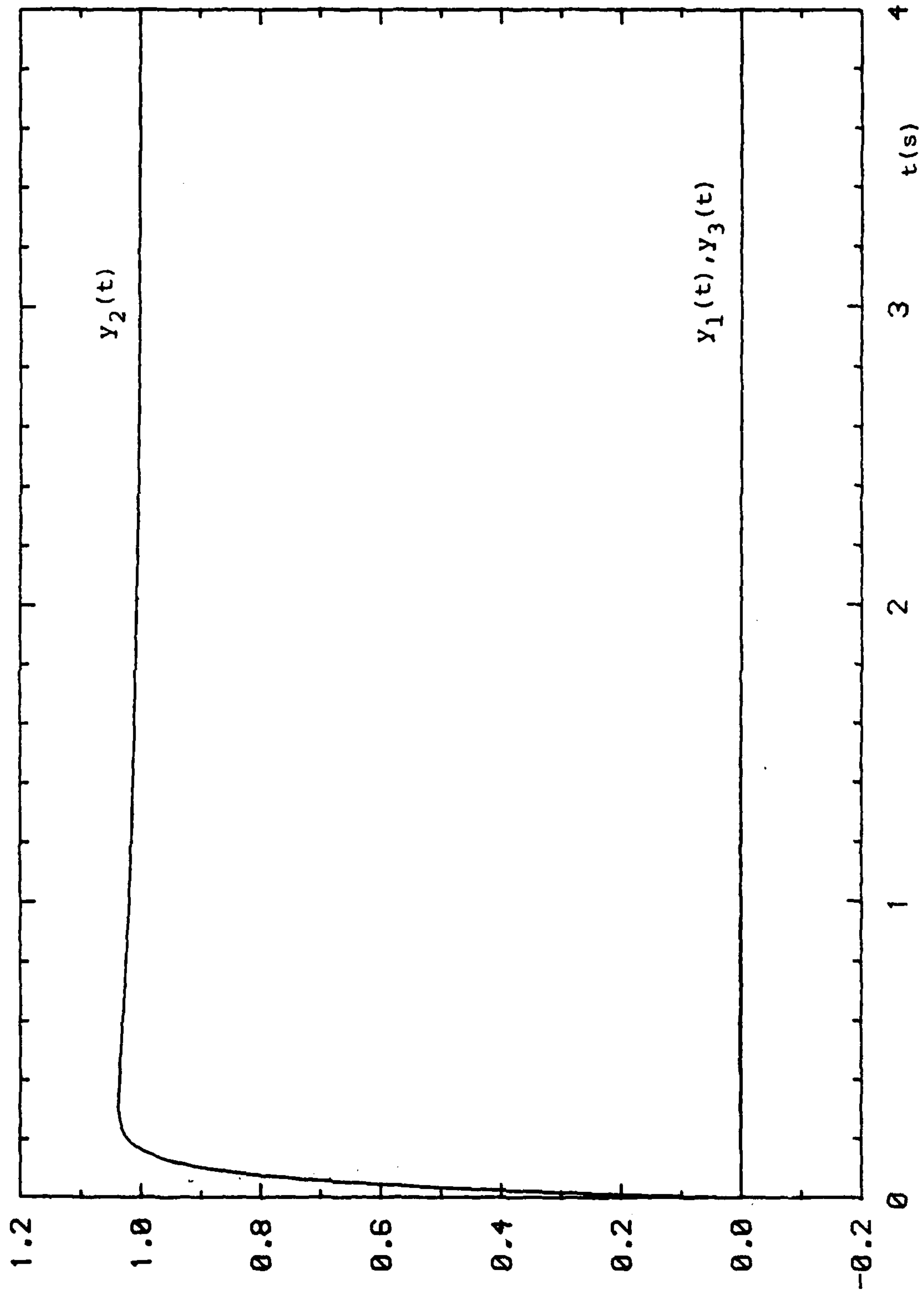

Fig 4.5(b) : g = 20

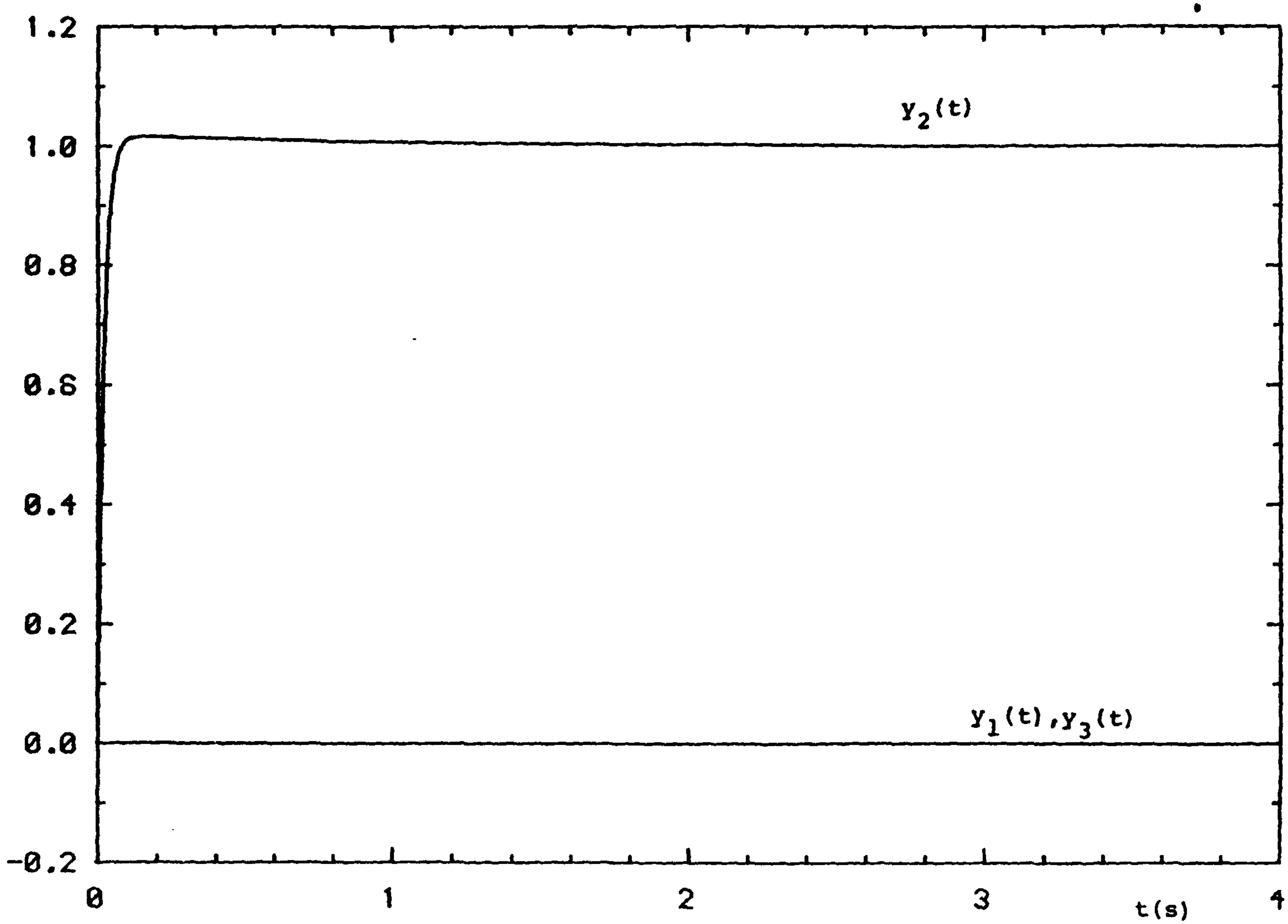

Fig 4.5(c): g = 50

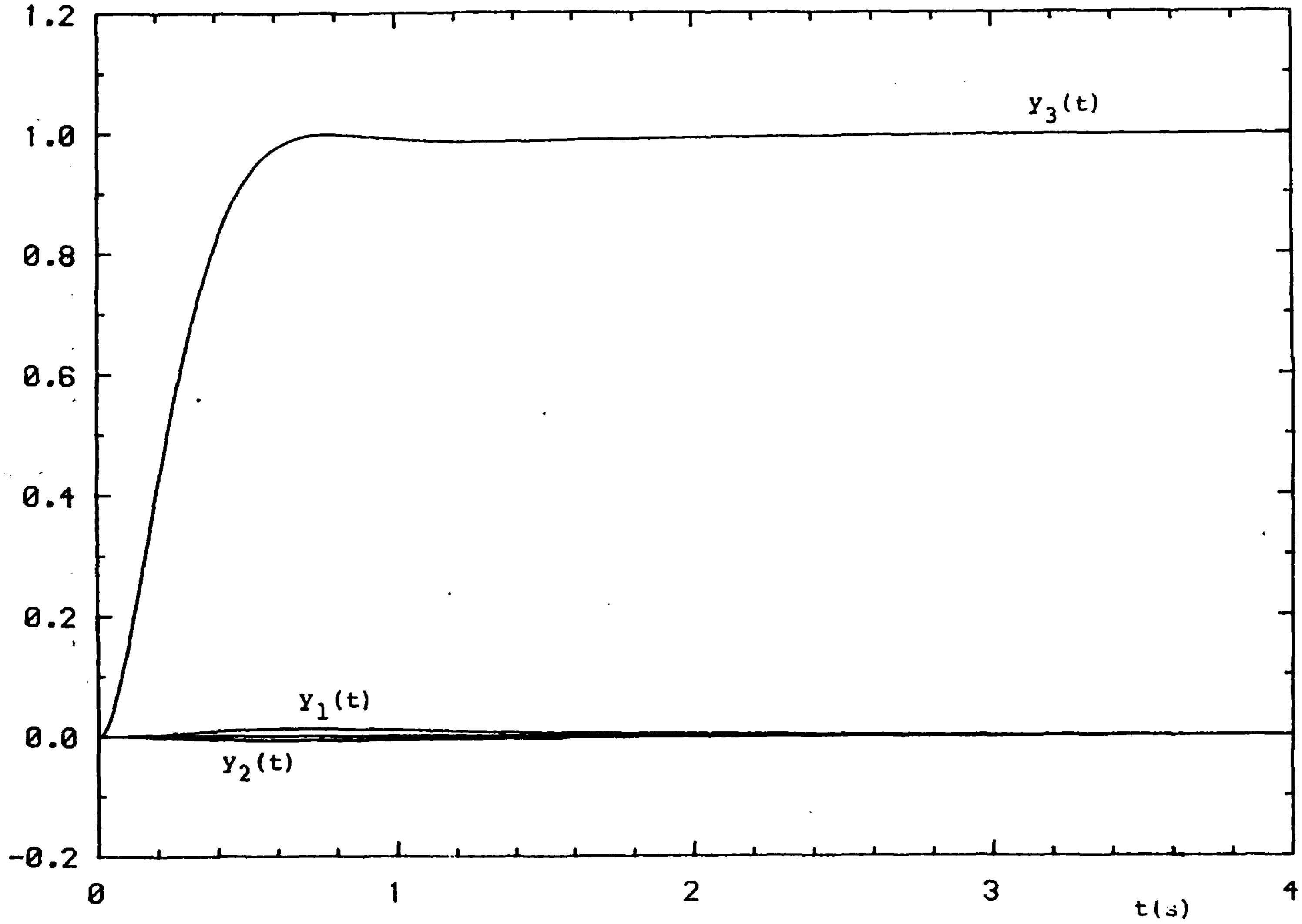

Fig 4.6(a): q = 10

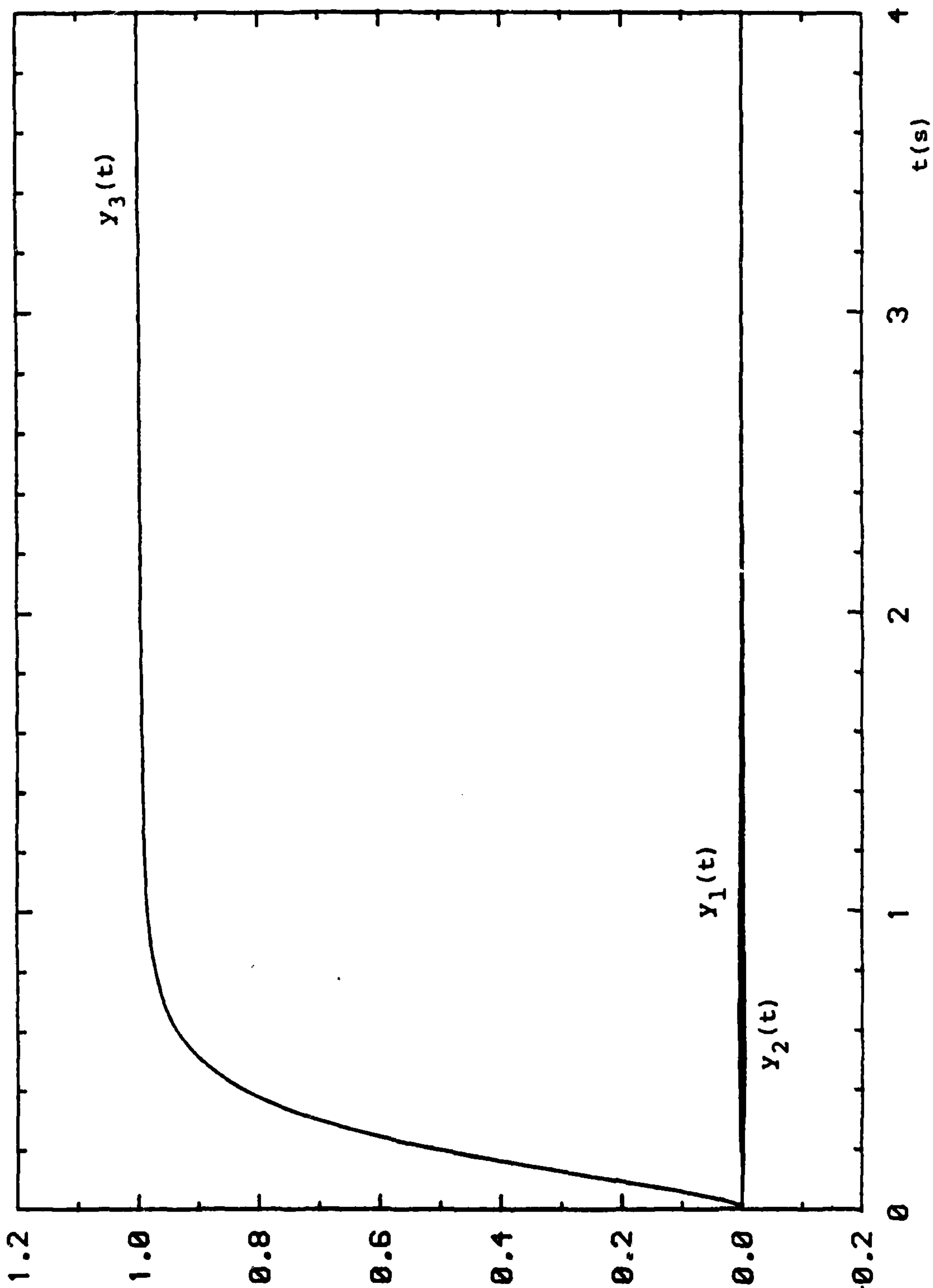

Fig 4.6(b): α = 20

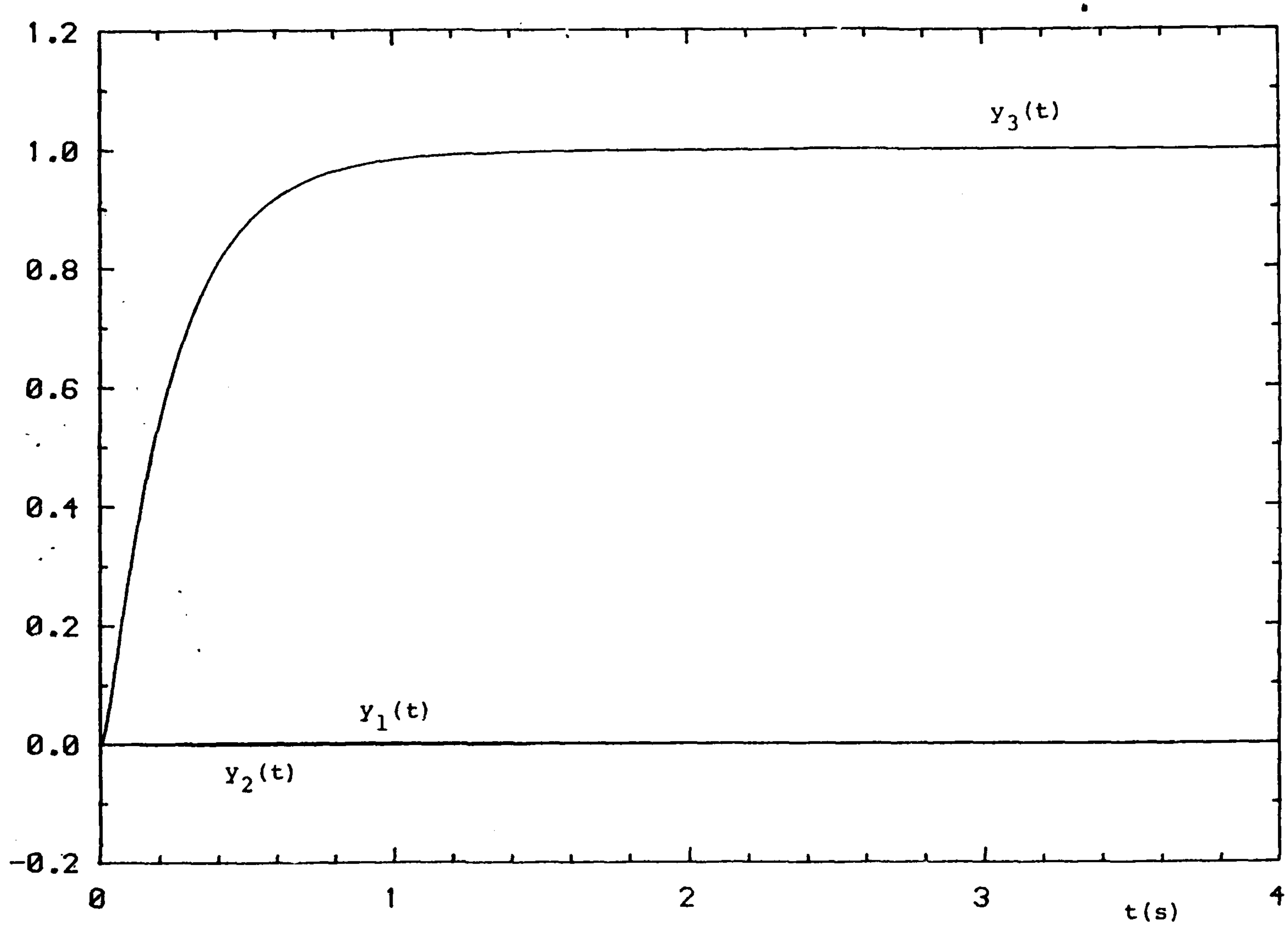

Fig 4.6(c): g = 50

CHAPTER 5

DESIGN OF TRACKING SYSTEMS INCORPORATING INNER-LOOP COMPENSATORS AND FAST-SAMPLING ERROR-ACTUATED CONTROLLERS

5.1 INTRODUCTION

In this chapter, singular perturbation methods are used to exhibit the asymptotic structure of the transfer function matrices of discrete-time tracking systems incorporating linear multivariable plants which are amenable to fast-sampling error-actuated digital control (Bradshaw and Porter 1980) only if extra plant output measurements are generated by the introduction of appropriate transducers and processed by inner-loop compensators. Such tracking systems consist of a linear multivariable plant governed on the continuous-time set $T = [0,+\infty)$ by state, output, and measurement equations of the respective forms

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} , & A_{12} \\ A_{21} , & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u(t) \quad , \qquad (5.1)$$

$$y(t) = [C_1 \ , \ C_2] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad , \qquad (5.2)$$

and

$$w(t) = [F_1 \ , \ F_2] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad , \qquad (5.3)$$

together with a fast-sampling error-actuated digital controller governed on the discrete-time set $T_T = \{0,T,2T,...\}$ by a control-law equation of the form

$$u(kT) = f\{K_0 e(kT)+K_1 z(kT)\} \qquad (5.4)$$

which is required to generate the control input vector

$u(t) = u(kT)$, $t \in [kT, (k+1)T)$, $kT \in T_T$, so as to cause the output vector $y(t)$ to track any constant command input vector $v(t)$ on T_T in the sense that

$$\lim_{k\to\infty}\{v(kT)-y(kT)\} = 0 \tag{5.5}$$

as a consequence of the fact that the error vector $e(t) = v(t) - w(t)$ assumes the steady-state value

$$\lim_{k\to\infty} e(kT) = \lim_{k\to\infty}\{v(kT)-w(kT)\} = 0 \tag{5.6}$$

for arbitrary initial conditions. In equations (5.1), (5.2), (5.3) and (5.4), $x_1(t) \in R^{n-\ell}$, $x_2(t) \in R^{\ell}$, $u(t) \in R^{\ell}$, $y(t) \in R^{\ell}$, $w(t) \in R^{\ell}$, $A_{11} \in R^{(n-\ell)x(n-\ell)}$, $A_{12} \in R^{(n-\ell)x\ell}$, $A_{21} \in R^{\ell x(n-\ell)}$, $A_{22} \in R^{\ell x\ell}$, $B_2 \in R^{\ell x\ell}$, $C_1 \in R^{\ell x(n-\ell)}$, $C_2 \in R^{\ell x\ell}$, $F_1 \in R^{\ell x(n-\ell)}$, $F_2 \in R^{\ell x\ell}$, rank $C_2B_2 < \ell$, rank $F_2B_2 = \ell$, $e(t) \in R^{\ell}$, $v(t) \in R^{\ell}$, $z(kT) = z(o) + T\sum_{j=0}^{j=k-1} e(jT) \in R^{\ell}$, $K_o \in R^{\ell x\ell}$, $K_1 \in R^{\ell x\ell}$, $f \in R^{+}$, and

$$[F_1 , F_2] = [C_1+MA_{11} , C_2+MA_{12}] \tag{5.7}$$

where $M \in R^{\ell x(n-\ell)}$. It is evident from equations (5.2), (5.3), and (5.7) that the vector

$$w(t) - y(t) = [MA_{11} , MA_{12}]\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{5.8}$$

of extra measurements is such that $v(kT)$ and $y(kT)$ satisfy the tracking condition (5.5) for any $M \in R^{\ell x(n-\ell)}$ if $e(kT)$ satisfies the steady-state condition (5.6) since equation (5.1) clearly implies that

$$\lim_{t\to\infty}[A_{11} \; , \; A_{12}]\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = 0 \tag{5.9}$$

in any steady state. However, the condition that rank $F_2B_2 = \ell$ requires that $C_2 \in R^{\ell x \ell}$ and $A_{12} \in R^{(n-\ell)x\ell}$ are such that $M \in R^{\ell x(n-\ell)}$ can be chosen so that

$$\text{rank } F_2 = \text{rank}(C_2+MA_{12}) = \ell \quad . \tag{5.10}$$

It is evident from equations (5.1),(5.2),(5.3), and (5.4) that such discrete-time tracking systems are governed on T_T by state and output equations of the respective forms

$$\begin{bmatrix} z\{(k+1)T\} \\ x_1\{(k+1)T\} \\ x_2\{(k+1)T\} \end{bmatrix}$$

$$= \begin{bmatrix} I_\ell \; , & -TF_1 \; , & -TF_2 \\ f\Psi_1K_1, & \Phi_{11}-f\Psi_1K_oF_1, & \Phi_{12}-f\Psi_1K_oF_2 \\ f\Psi_2K_1, & \Phi_{21}-f\Psi_2K_oF_1, & \Phi_{22}-f\Psi_2K_oF_2 \end{bmatrix}\begin{bmatrix} z(kT) \\ x_1(kT) \\ x_2(kT) \end{bmatrix}$$

$$+ \begin{bmatrix} TI_\ell \\ f\Psi_1K_o \\ f\Psi_2K_o \end{bmatrix} v(kT) \tag{5.11}$$

and

$$y(kT) = [0 \; , \; C_1 \; , \; C_2]\begin{bmatrix} z(kT) \\ x_1(kT) \\ x_2(kT) \end{bmatrix} \tag{5.12}$$

where

$$\begin{bmatrix}\Phi_{11}, & \Phi_{12}\\ \Phi_{21}, & \Phi_{22}\end{bmatrix} = \exp\left\{\begin{bmatrix}A_{11}, & A_{12}\\ A_{21}, & A_{22}\end{bmatrix}T\right\} \tag{5.13}$$

and

$$\begin{bmatrix}\Psi_1\\ \Psi_2\end{bmatrix} = \int_0^T \exp\left\{\begin{bmatrix}A_{11}, & A_{12}\\ A_{21}, & A_{22}\end{bmatrix}t\right\}\begin{bmatrix}0\\ B_2\end{bmatrix}dt \quad . \tag{5.14}$$

The transfer function matrix relating the plant output vector to the command input vector of the closed-loop discrete-time tracking system governed by equations (5.11) and (5.12) is clearly

$G(\lambda)$ =

$$[0, C_1, C_2]\begin{bmatrix}\lambda I_\ell - I_\ell, & TF_1 & , & TF_2\\ -f\Psi_1K_1, & \lambda I_{n-\ell}-\Phi_{11}+f\Psi_1K_oF_1, & -\Phi_{12}+f\Psi_1K_oF_2\\ -f\Psi_2K_1, & -\Phi_{21}+f\Psi_2K_oF_1 & , \lambda I_\ell-\Phi_{22}+f\Psi_2K_oF_2\end{bmatrix}^{-1}\begin{bmatrix}TI_\ell\\ f\Psi_1K_o\\ f\Psi_2K_o\end{bmatrix} \tag{5.15}$$

and the fast-sampling tracking characteristics of this system can accordingly be elucidated by invoking the results of Porter and Shenton (1975) from the singular perturbation analysis of transfer function matrices.

These results yield the asymptotic form of $G(\lambda)$ as the sampling frequency $f \to \infty$ and thus greatly facilitate the determination of controller and transducer matrices K_o, K_1, and M such that the discrete-time tracking behaviour of the closed-loop system becomes increasingly non-interacting as f is increased. The frequency-response and step-response characteristics of a discrete-time flight-control system for the longitudinal dynamics of an aircraft (Kouvaritakis, Murray, and MacFarlane 1979) are presented in order to illustrate these general results.

5.2 ANALYSIS

It is evident from equation (5.15) that, by regarding $\varepsilon = 1/f$ as the perturbation parameter, the asymptotic form of the transfer function matrix $G(\lambda)$ of the discrete-time tracking system as $f \to \infty$ can be determined by invoking the results of Porter and Shenton (1975) from the ıngular perturbation analysis of transfer function matrices. Indeed, since it follows from equations (5.13) and (5.14) that

$$\lim_{f\to\infty} f\begin{bmatrix} \Phi_{11}-I_{n-\ell} \, , & \Phi_{12} \\ \Phi_{21} \, , & \Phi_{22}-I_{\ell} \end{bmatrix} = \begin{bmatrix} A_{11} \, , & A_{12} \\ A_{21} \, , & A_{22} \end{bmatrix} \tag{5.16}$$

and

$$\lim_{f\to\infty} f\begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \quad , \tag{5.17}$$

these results indicate that as $f \to \infty$ the transfer function matrix $G(\lambda)$ assumes the asymptotic form

$$\Gamma(\lambda) = \tilde{\Gamma}(\lambda) + \hat{\Gamma}(\lambda) \tag{5.18}$$

where

$$\tilde{\Gamma}(\lambda) = C_o(\lambda I_n - I_n - TA_o)^{-1}TB_o \quad , \tag{5.19}$$

$$\hat{\Gamma}(\lambda) = C_2(\lambda I_\ell - I_\ell - A_4)^{-1}B_2K_o \quad , \tag{5.20}$$

$$A_o = \begin{bmatrix} -K_o^{-1}K_1 & , & 0 \\ A_{12}F_2^{-1}K_o^{-1}K_1 & , & A_{11}-A_{12}F_2^{-1}F_1 \end{bmatrix} \quad , \tag{5.21}$$

$$B_o = \begin{bmatrix} 0 \\ A_{12}F_2^{-1} \end{bmatrix} \quad , \tag{5.22}$$

$$C_o = [C_2F_2^{-1}K_o^{-1}K_1 \ , \ C_1-C_2F_2^{-1}F_1] \quad , \tag{5.23}$$

and

$$A_4 = -B_2K_oF_2 \quad . \tag{5.24}$$

It follows from equations (5.18), (5.19), and (5.21) that the 'slow' modes Z_s of the tracking system correspond as $f \to \infty$ to the poles $Z_1 \cup Z_2$ of $\tilde{\Gamma}(\lambda)$ where

$$Z_1 = \{\lambda \in C: |\lambda I_\ell - I_\ell + TK_o^{-1}K_1| = 0\} \tag{5.25}$$

and

$$Z_2 = \{\lambda \in C: |\lambda I_{n-\ell} - I_{n-\ell} - TA_{11} + TA_{12}F_2^{-1}F_1| = 0\} \quad , \tag{5.26}$$

and from equations (5.8), (5.20), and (5.24) that the 'fast' modes Z_f of the tracking system correspond as $f \to \infty$ to the poles Z_3 of $\hat{\Gamma}(\lambda)$ where

$$Z_3 = \{\lambda \in C: |\lambda I_\ell - I_\ell + F_2B_2K_o| = 0\} \quad . \tag{5.27}$$

Furthermore, it follows from equations (5.19), (5.21), (5.22), and (5.23) that the 'slow' transfer function matrix

$$\tilde{\Gamma}(\lambda) = (C_1 - C_2F_2^{-1}F_1)(\lambda I_{n-\ell} - I_{n-\ell} - TA_{11} + TA_{12}F_2^{-1}F_1)^{-1}TA_{12}F_2^{-1} \tag{5.28}$$

and from equations (5.20) and (5.24) that the 'fast' transfer function matrix

$$\hat{\Gamma}(\lambda) = C_2F_2^{-1}(\lambda I_\ell - I_\ell + F_2B_2K_o)^{-1}F_2B_2K_o \quad . \tag{5.29}$$

Hence, in view of equations (5.28) and (5.29), it is evident from equation (5.18) that as $f \to \infty$ the transfer function matrix $G(\lambda)$ of the discrete-time tracking system assumes the asymptotic form

$$\Gamma(\lambda) = (C_1 - C_2F_2^{-1}F_1)(\lambda I_{n-\ell} - I_{n-\ell} - TA_{11} + TA_{12}F_2^{-1}F_1)^{-1}TA_{12}F_2^{-1}$$
$$+ C_2F_2^{-1}(\lambda I_\ell - I_\ell + F_2B_2K_o)^{-1}F_2B_2K_o \tag{5.30}$$

in consonance with the fact that both the 'slow' modes corresponding to the poles Z_2 and the 'fast' modes corresponding to the poles Z_3 possibly remain both controllable and observable as $f \to \infty$. However, the 'slow' transfer function matrix $\tilde{\Gamma}(\lambda)$ reduces to the form expressed by equation (5.28) precisely because the 'slow' modes corresponding to the poles Z_1 definitely become asymptotically uncontrollable as $f \to \infty$ in view of the block structure of the matrices A_o and B_o in equations (5.21) and (5.22).

5.3 SYNTHESIS

It is evident from equations (5.7), (5.9), (5.11), and (5.12) that tracking will occur in the sense of equation (5.5) provided only that

$$Z_s \cup Z_f \subset \mathcal{D}^- \tag{5.31}$$

where $\mathcal{D}^-$ is the open unit disc. In view of equations (5.25), (5.26) and (5.27), the 'slow' and 'fast' modes will satisfy the tracking requirement (5.31) for sufficiently small sampling periods if the controller and transducer matrices K_o, K_1, and M are chosen such that both $Z_1 \subset \mathcal{D}^-$ and $Z_2 \subset \mathcal{D}^-$ for sufficiently small sampling periods and $Z_3 \subset \mathcal{D}^-$ in the case of plants for which the transducer matrix M can be simultaneously chosen so as to satisfy the measurement condition expressed by equation (5.10). Moreover, if K_o and M are chosen so that both $\tilde{\Gamma}(\lambda)$ and $\hat{\Gamma}(\lambda)$ are diagonal transfer function matrices by requiring that

$$F_2B_2K_o = (C_2+MA_{12})B_2K_o = \mathrm{diag}\{\sigma_1,\sigma_2,\ldots,\sigma_\ell\} \quad (5.32)$$

where $1-\sigma_j \in \mathcal{R} \cap \mathcal{D}^-$ $(j=1,2,\ldots,\ell)$ in the case of $\hat{\Gamma}(\lambda)$, it follows from equation (5.24) that the transfer function matrix $G(\lambda)$ of the discrete-time tracking system will assume the diagonal asymptotic form $\Gamma(\lambda)$ and therefore that increasingly non-interacting tracking behaviour will occur as $f \to \infty$. Furthermore, such tracking behaviour will exhibit high accuracy in the face of plant-parameter variations provided that the steady-state conditions expressed by equation (5.9) correspond to 'kinematic' relationships which hold between the state variables as a consequence of the fundamental dynamical structure of the plant.

However, increasingly 'tight' tracking behaviour will not in general occur as $f \to \infty$ in view of the possible presence in the plant output vector of 'slow' modes corresponding to the poles Z_2 of the 'slow' transfer function matrix $\tilde{\Gamma}(\lambda)$. But the presence of any such 'slow' modes is the inevitable consequence of the introduction of appropriate transducers which generate extra plant output measurements so as to ensure that rank $F_2B_2 = \ell$ and thus render plants for which rank $C_2B_2 < \ell$ amenable to fast-sampling error-actuated control. Indeed, increasingly 'tight' tracking behaviour is achievable as $f \to \infty$ only in the case of plants for which rank $C_2B_2 = \ell$ (Bradshaw and Porter 1980) and which are accordingly amenable to fast-sampling error-actuated control without the necessity for the generation of extra plant output measurements.

5.4 ILLUSTRATIVE EXAMPLE

These general results can be conveniently illustrated by designing a fast-sampling error-actuated digital flight controller for the longitudinal dynamics of an aircraft governed on T by the respective state and output equations (Kouvaritakis, Murray, and MacFarlane 1979)

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \\ \dot{x}_5(t) \end{bmatrix} = \begin{bmatrix} 0, & 1.132, & 0, & 0, & 1 \\ 0, & 0, & 0, & 1, & 0 \\ 0, & -0.1712, & -0.0538, & 0, & 0.0705 \\ 0, & 0, & 0.0485, & -0.8536, & -1.013 \\ 0, & 0, & -0.2909, & 1.0532, & -0.6859 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix}$$

$$+ \begin{bmatrix} 0, & 0, & 0 \\ 0, & 0, & 0 \\ -0.0012, & 1, & 0 \\ 0.4419, & 0, & -1.6646 \\ 0.1575, & 0, & -0.0732 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} \qquad (5.33)$$

and

$$\begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = \begin{bmatrix} 1, & 0, & 0, & 0, & 0 \\ 0, & 0, & 1, & 0, & 0 \\ 0, & 1, & 0, & 0, & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix} \tag{5.34}$$

from which it can be readily verified that the first Markov parameter

$$C_2B_2 = \begin{bmatrix} 0 & , 0 & , 0 \\ -0.0012 & , 1 & , 0 \\ 0 & , 0 & , 0 \end{bmatrix} \tag{5.35}$$

is rank defective. In case $\{\sigma_1,\sigma_2,\sigma_3\} = \{1,1,1\}$, $K_1 = K_o$, and

$$M = \begin{bmatrix} 0.25 & , 0 \\ 0 & , 0 \\ 0 & , 0.25 \end{bmatrix} \quad , \tag{5.36}$$

it follows from equations (5.3), (5.4), and (5.32) that the corresponding transducers and fast-sampling error-actuated digital controller are governed on T and T_T by the respective measurement and control-law equations

$$\begin{bmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \end{bmatrix} = \begin{bmatrix} 1, & 0.283, & 0, & 0 & , 0.25 \\ 0, & 0 & , 1, & 0 & , 0 \\ 0, & 1 & , 0, & 0.25, & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix} \tag{5.37}$$

and

$$\begin{bmatrix} u_1(kT) \\ u_2(kT) \\ u_3(kT) \end{bmatrix} = f\left\{\begin{bmatrix} -28.97 & , 0 , & -1.274 \\ -0.0348 & , 1 , & -0.0015 \\ -7.690 & , 0 , & -2.740 \end{bmatrix}\begin{bmatrix} e_1(kT) \\ e_2(kT) \\ e_3(kT) \end{bmatrix} + \begin{bmatrix} -28.97 & , 0 , & -1.274 \\ -0.0348 & , 1 , & -0.0015 \\ -7.690 & , 0 , & -2.740 \end{bmatrix}\begin{bmatrix} z_1(kT) \\ z_2(kT) \\ z_3(kT) \end{bmatrix}\right\} \quad (5.38)$$

and it is evident from equations (5.25),(5.26),and(5.27) that $Z_1 = \{1-T, 1-T, 1-T\}$, $Z_2 = \{1-4T, 1-4T, 1-4T\}$, and $Z_3 = \{0,0,0\}$. It is also evident from equation (5.30) that the asymptotic transfer function matrix assumes the diagonal form

$$\Gamma(\lambda) = \begin{bmatrix} \frac{4T}{\lambda-1+4T} & , 0 , & 0 \\ 0 & , \frac{1}{\lambda} , & 0 \\ 0 & , 0 , & \frac{4T}{\lambda-1+4T} \end{bmatrix} \quad (5.39)$$

and therefore that the closed-loop discrete-time flight-control system for the longitudinal dynamics of the aircraft will exhibit increasingly non-interacting tracking behaviour as $f \to \infty$ when the piecewise-constant control input vector $[u_1(t) , u_2(t) , u_3(t)]^T = [u_1(kT) , u_2(kT) , u_3(kT)]^T$, $t \in [kT , (k+1)T)$, $kT \in T_T$, is generated by the fast-sampling digital controller governed on T_T by equation (5.38). However, it is apparent from equation (5.35) that increasingly 'tight' tracking behaviour will be achieved only in the case of $y_2(t)$ in view of the presence in $y_1(t)$ and $y_3(t)$ of the 'slow' modes corresponding to the poles Z_2.

The actual frequency-response loci $G(e^{i\omega T})$ for $\omega T \in [0 , 2\pi]$ are shown in Figs 5.1, 5.2, and 5.3 when $T = 0.10, 0.05$, and 0.02, respectively, and it is clear that the actual frequency-response loci approach the asymptotic frequency-response loci $\Gamma(e^{i\omega T})$ as the sampling frequency f is increased. The corresponding step-response characteristics are shown in Fig 5.4, Fig 5.5, and Fig 5.6 when $[v_1(t) , v_2(t) , v_3(t)]^T = [1 , 0 , 0]^T$, $[v_1(t) , v_2(t) , v_3(t)]^T = [0 , 1 , 0]^T$, and $[v_1(t) , v_2(t) , v_3(t)]^T = [0 , 0 , 1]^T$, respectively, and it is evident that increasingly non-interacting tracking occurs as the sampling frequency f is increased but that increasingly 'tight' tracking occurs only in the case of $y_2(t)$.

5.5 Conclusion

Singular perturbation methods have been used to exhibit the asymptotic structure of the transfer function matrices of discrete-time tracking systems incorporating linear multivariable plants which are amenable to fast-sampling error-actuated digital control only if extra plant output measurements are generated by the introduction of appropriate transducers and processed by inner-loop compensators. It has been shown that these results greatly facilitate the determination of controller and transducer matrices which ensure that the closed-loop behaviour of such discrete-time tracking systems becomes increasingly non-interacting as the sampling frequency f is increased. These general results have been illustrated by the presentation of the frequency-response and step-response characteristics of a discrete-time flight-control system for the longitudinal dynamics of an aircraft.

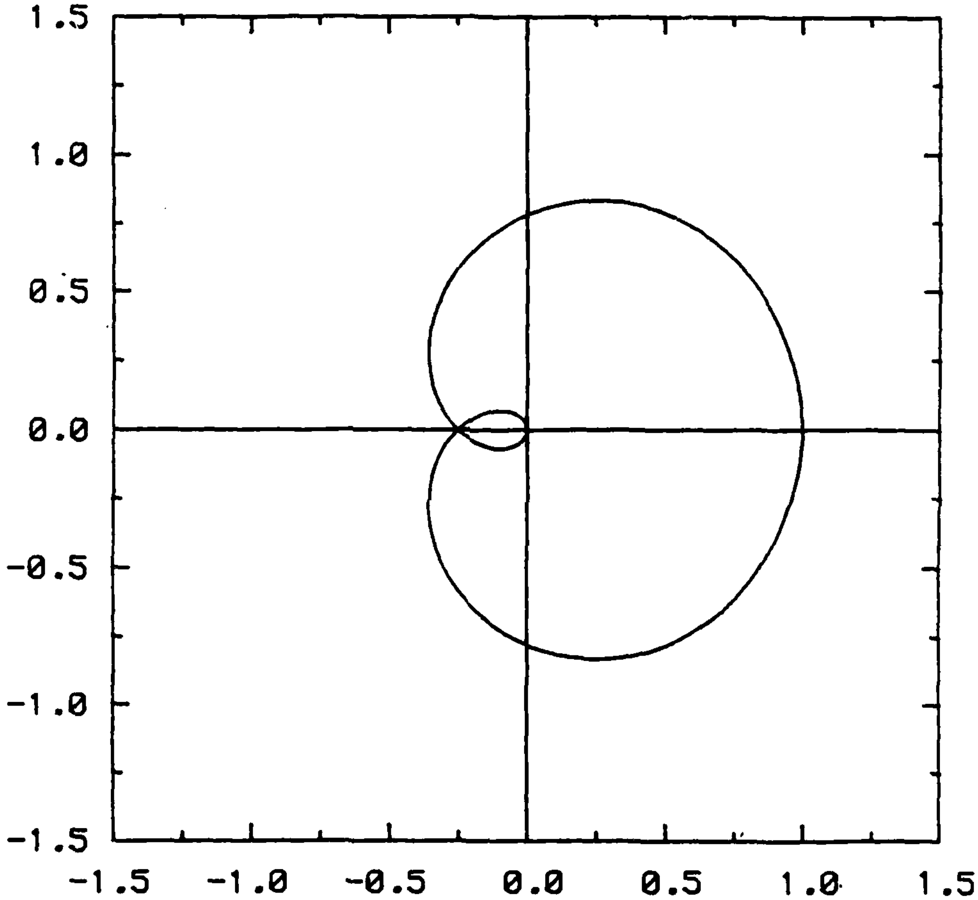

Fig 5.1(a): $G_{11}(e^{i\omega T})$; T = 0.10

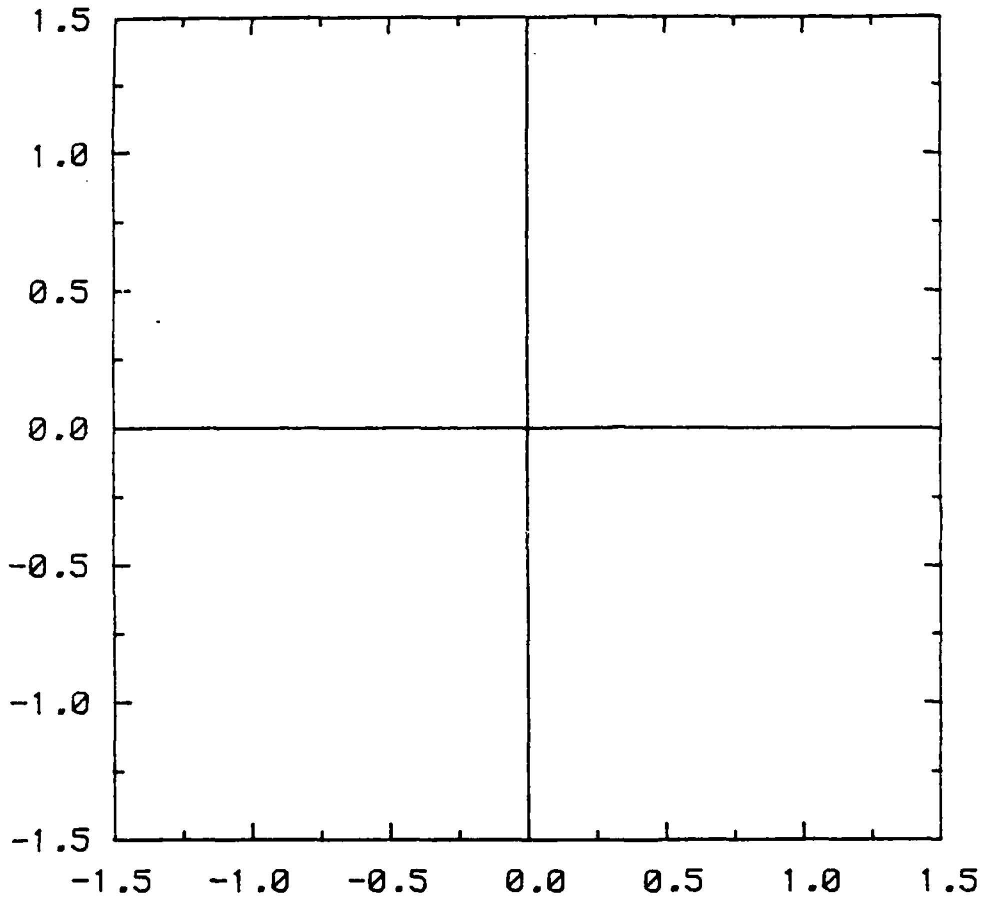

Fig 5.1(b): $G_{12}(e^{i\omega T})$; T = 0.10

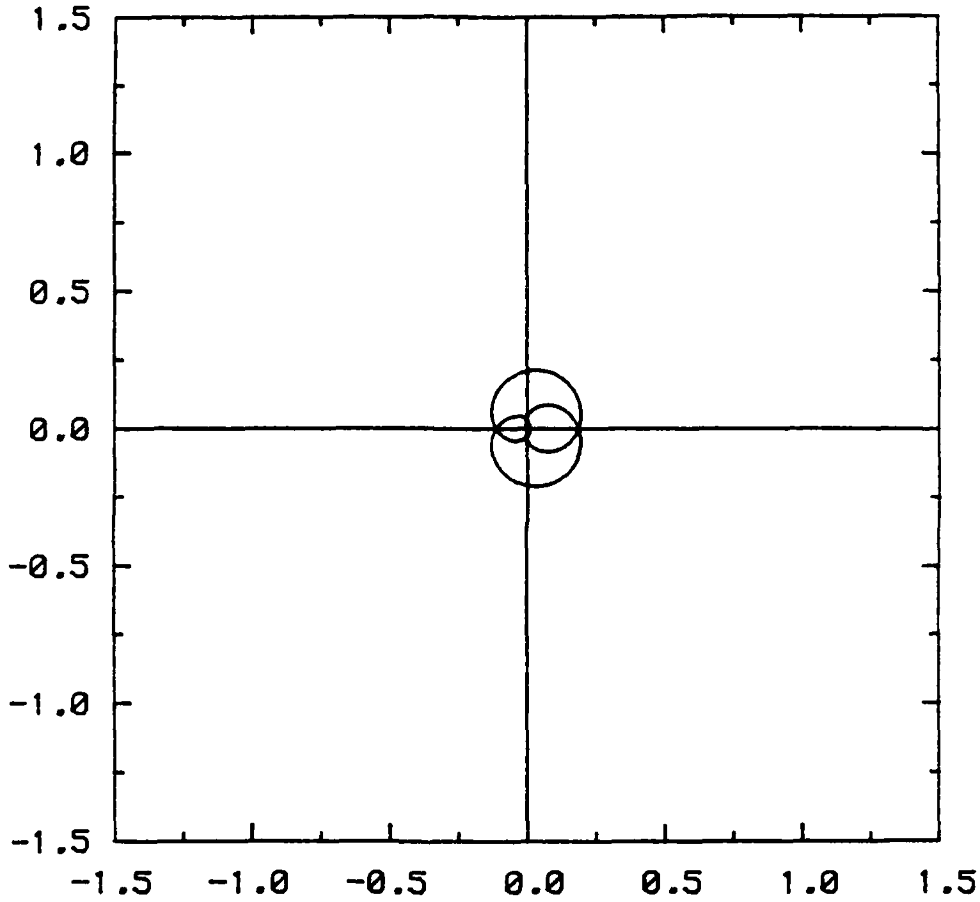

Fig 5.1(c): $G_{13}(e^{i\omega T})$; T = 0.10

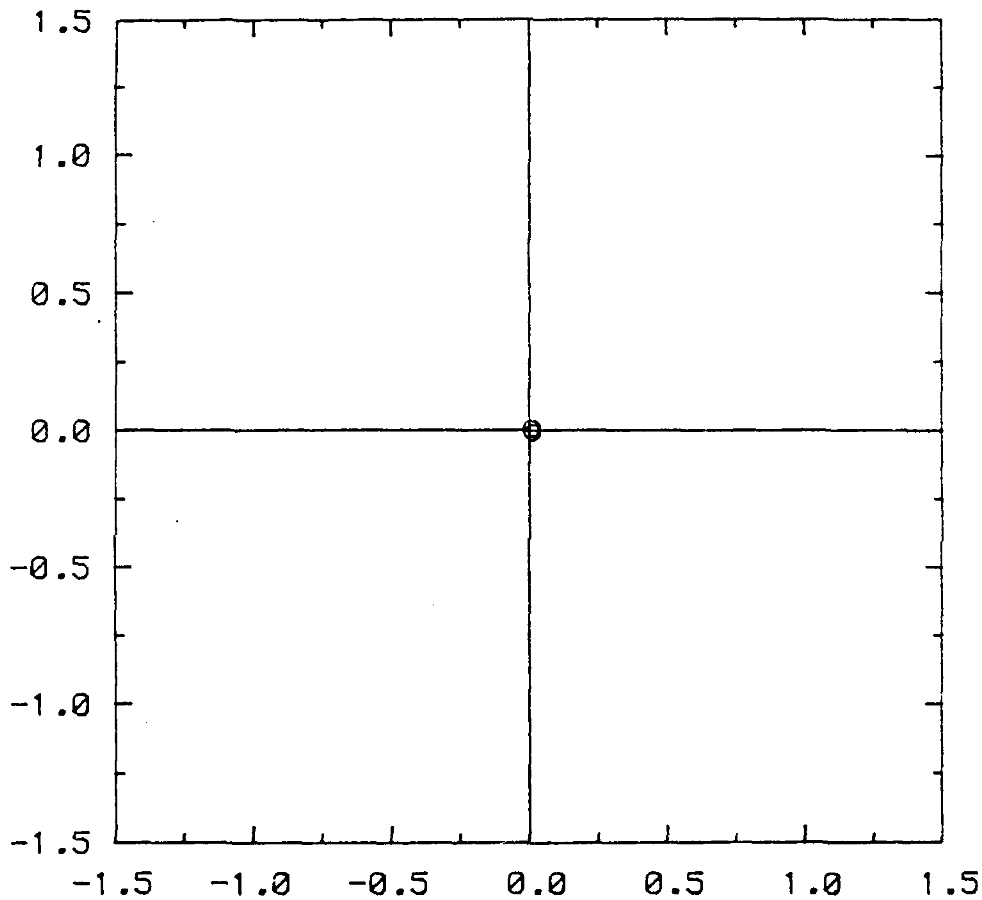

Fig 5.1(d): $G_{21}(e^{i\omega T})$; T = 0.10

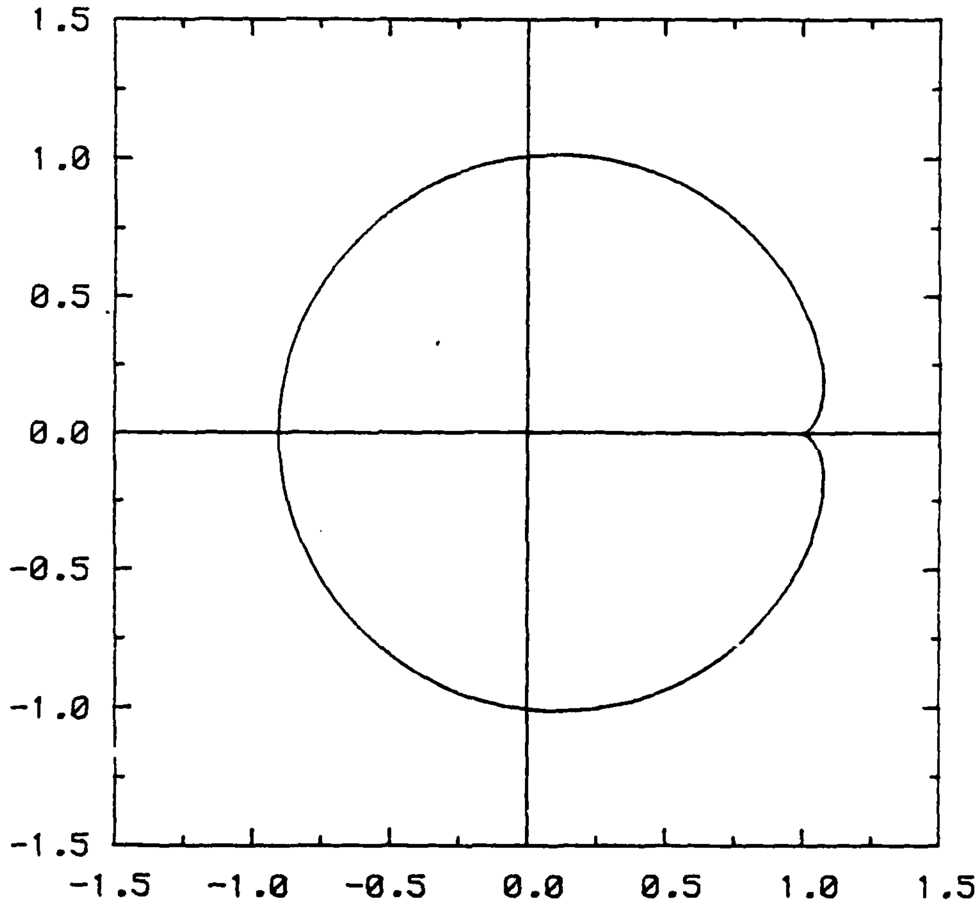

Fig 5.1(e): $G_{22}(e^{i\omega T})$; T = 0.10

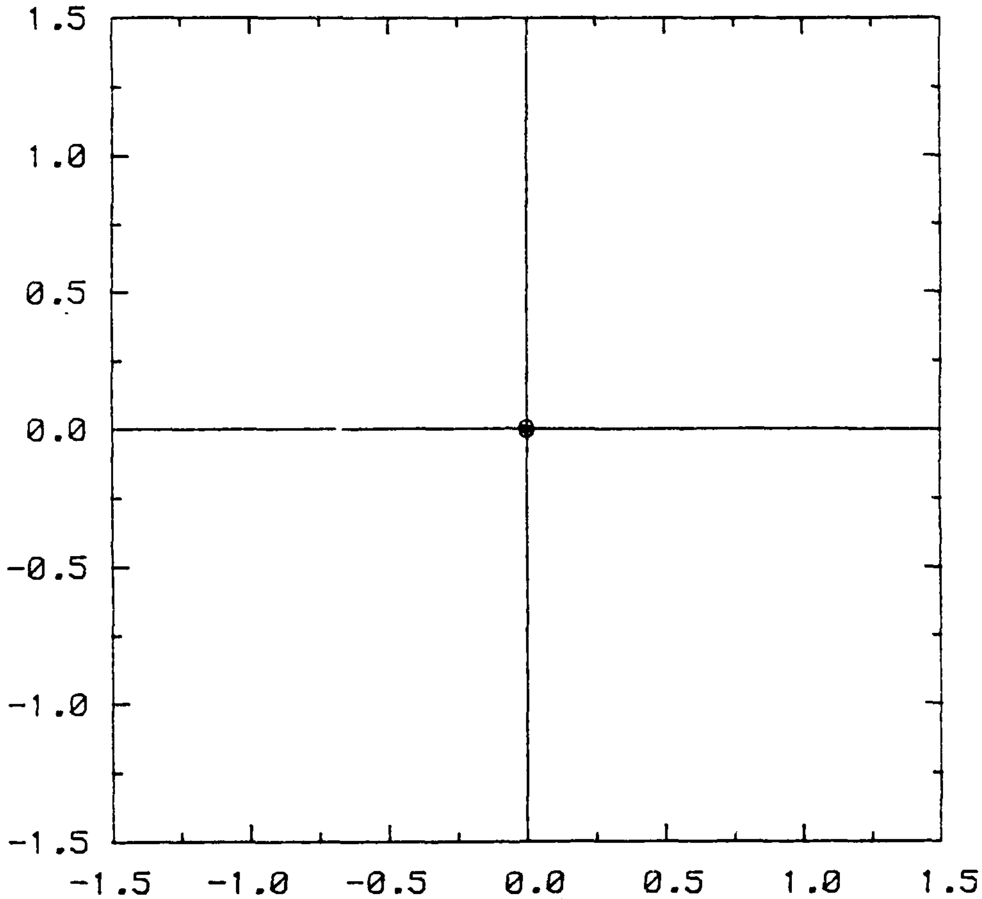

Fig 5.1(f): $G_{23}(e^{i\omega T})$; T = 0.10

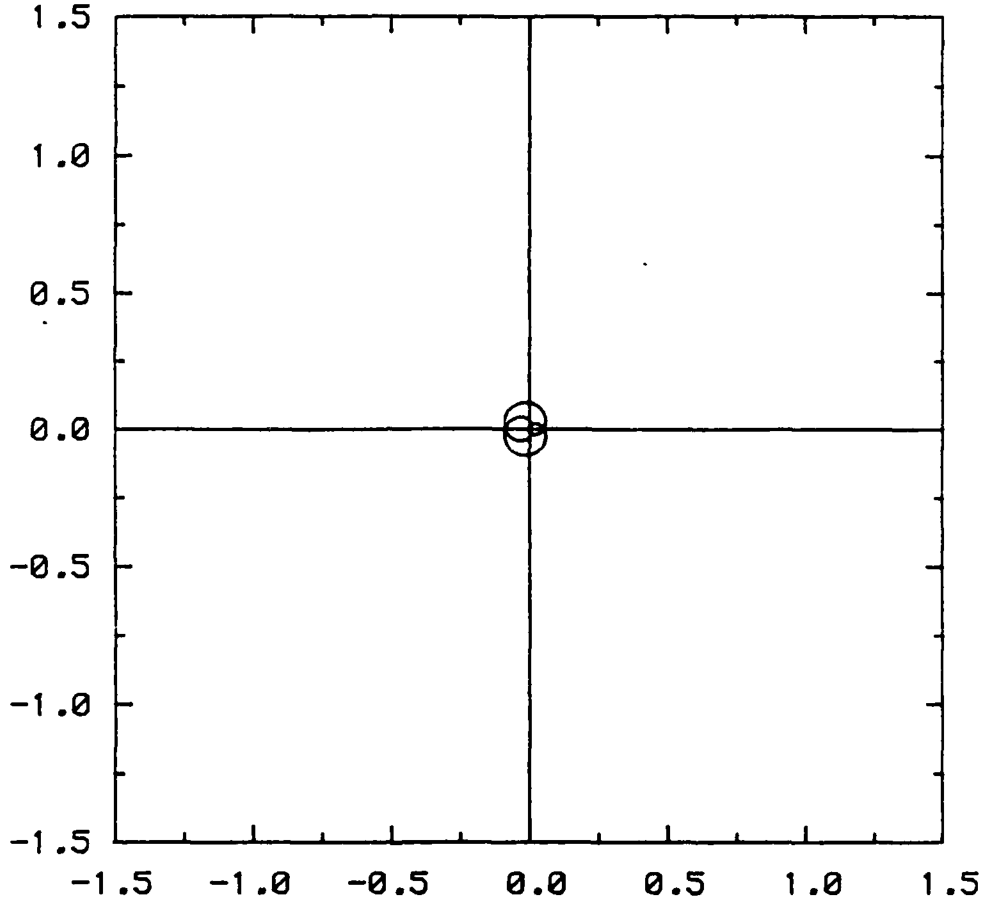

Fig 5.1(g): $G_{31}(e^{i\omega T})$; T = 0.10

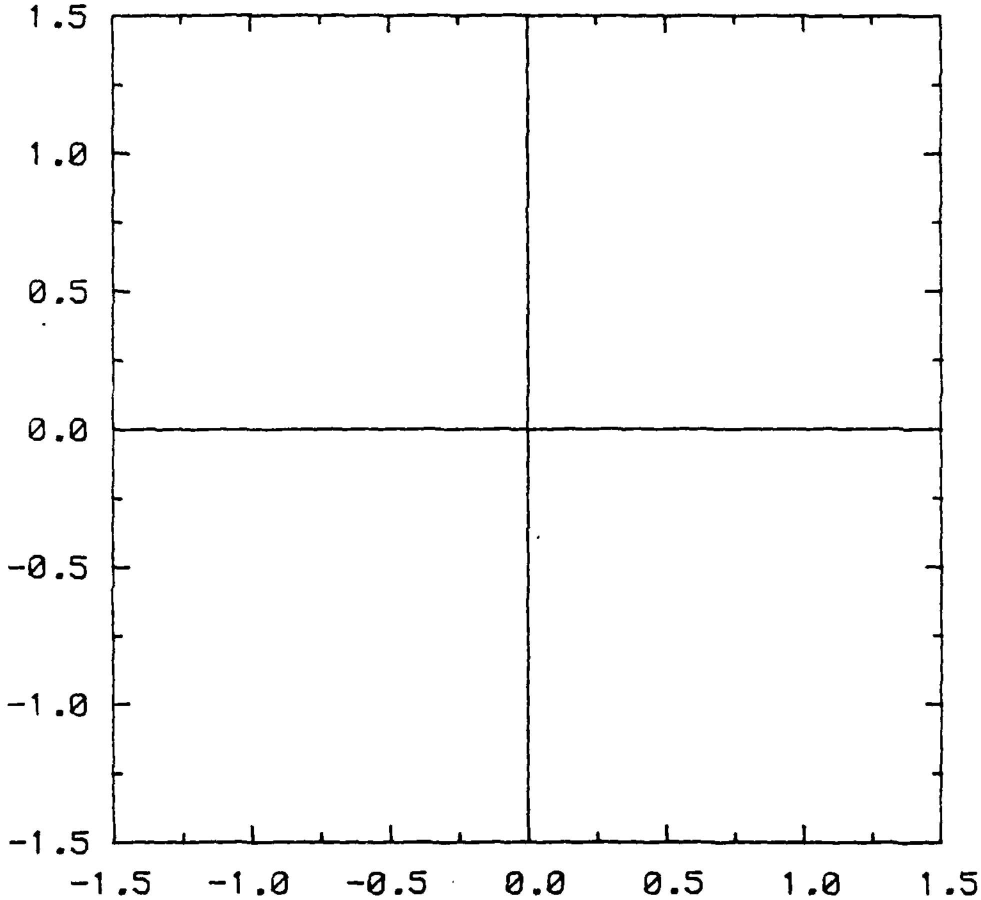

Fig 5.1(h): $G_{32}(e^{i\omega T})$; T = 0.10

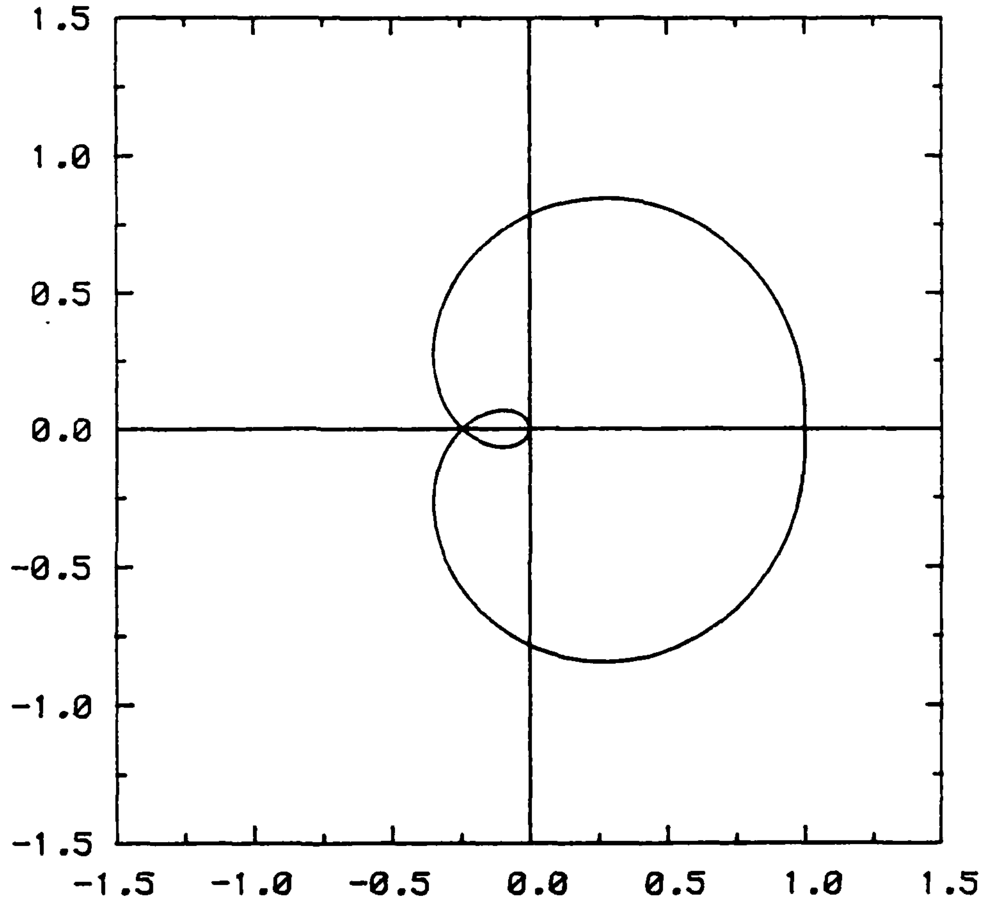

Fig 5.1(i): $G_{33}(e^{i\omega T})$; T = 0.10

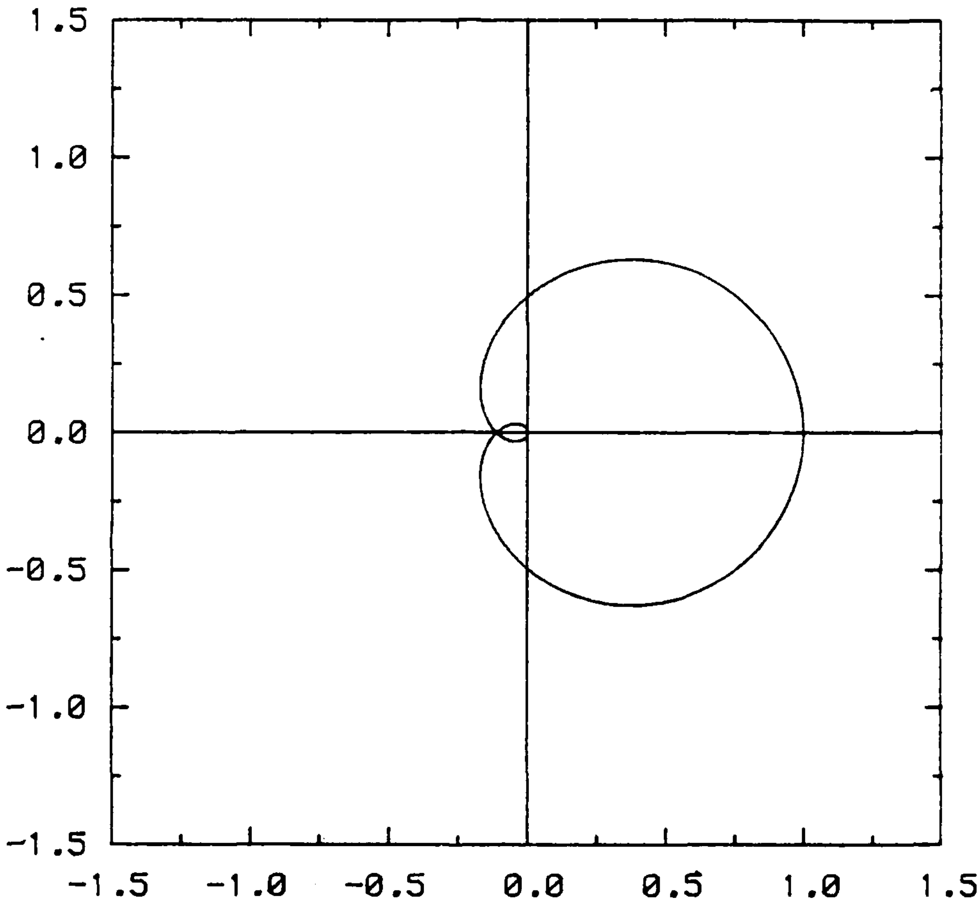

Fig 5.2(a): $G_{11}(e^{i\omega T})$; $T = 0.05$

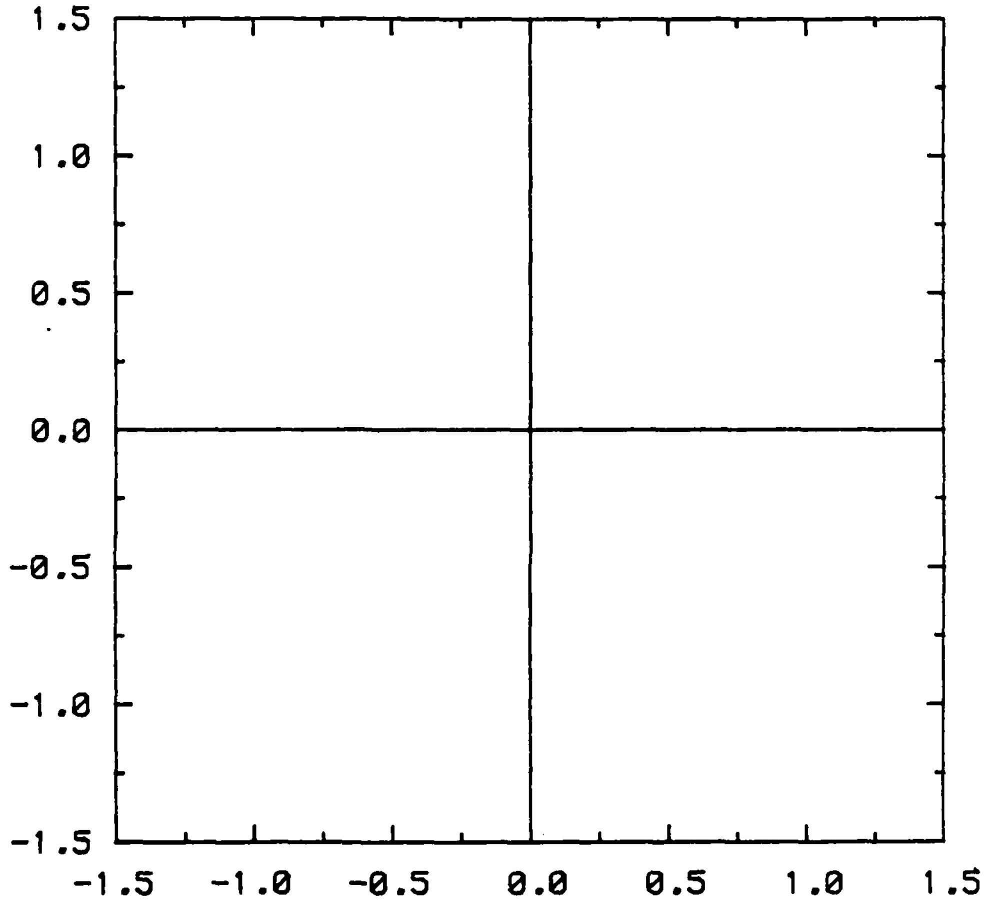

Fig 5.2(b): $G_{12}(e^{i\omega T})$; T = 0.05

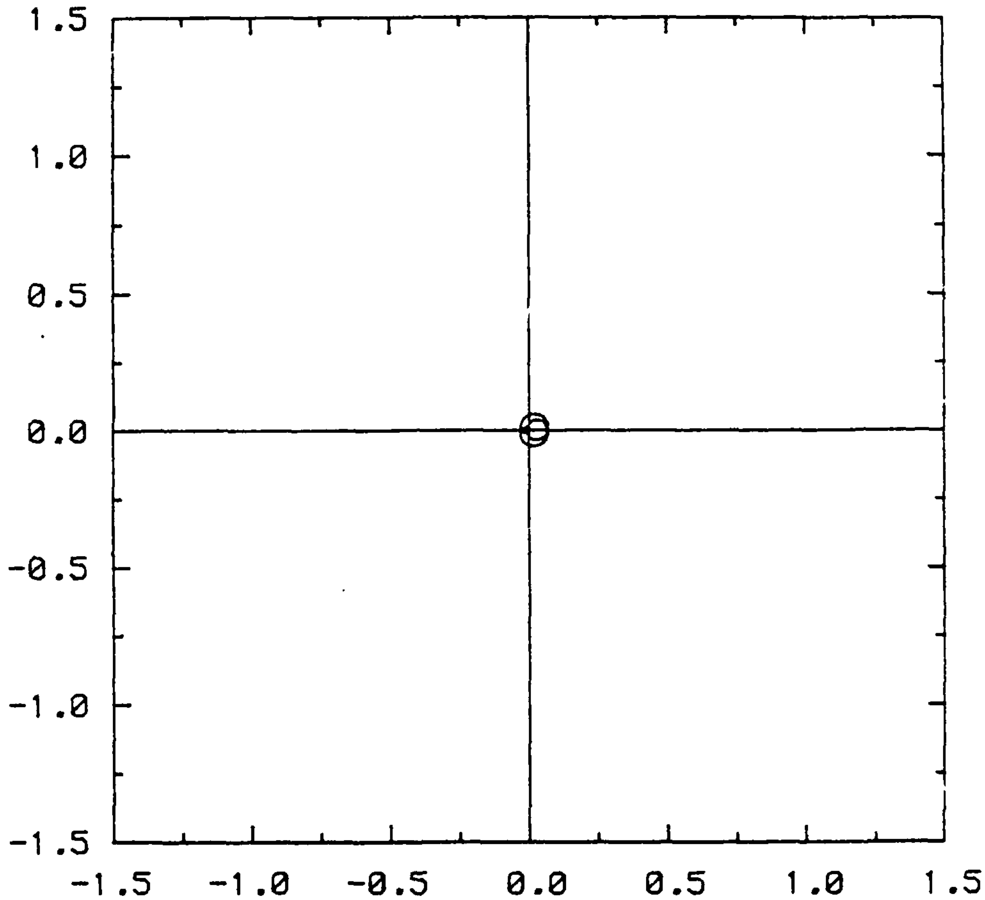

Fig 5.2(c): $G_{13}(e^{i\omega T})$; T = 0.05

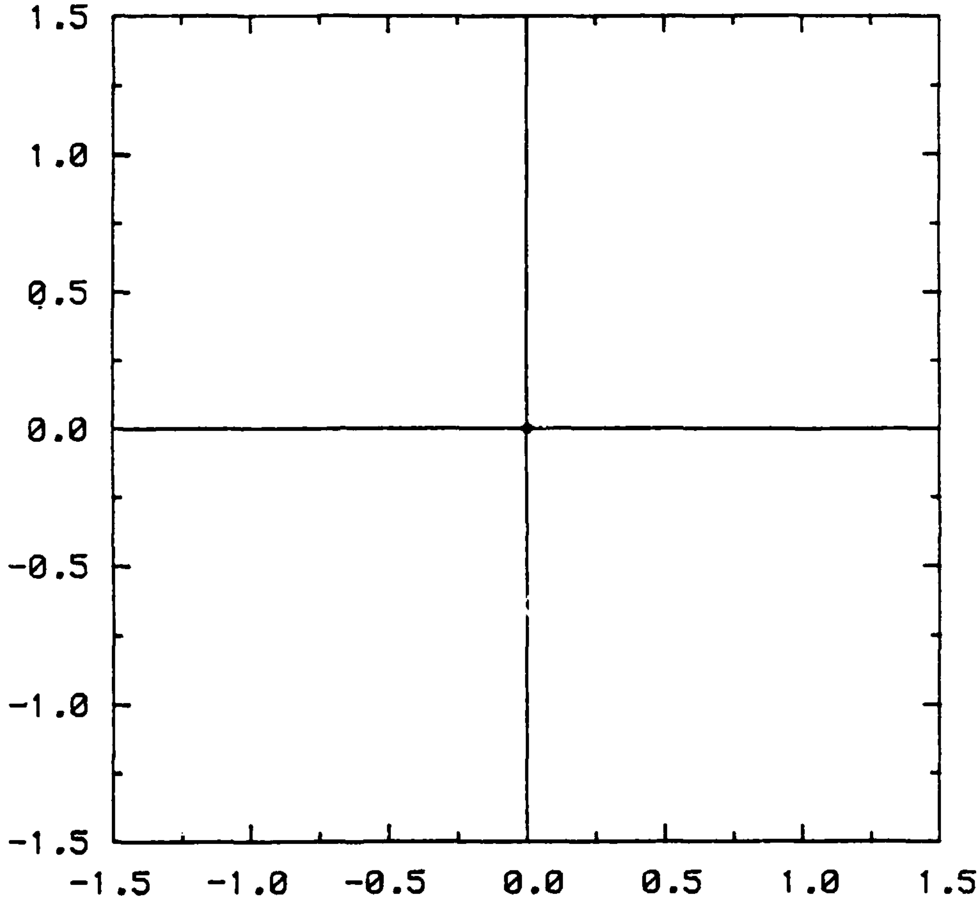

Fig 5.2(d): $G_{21}(e^{i\omega T})$; T = 0.05

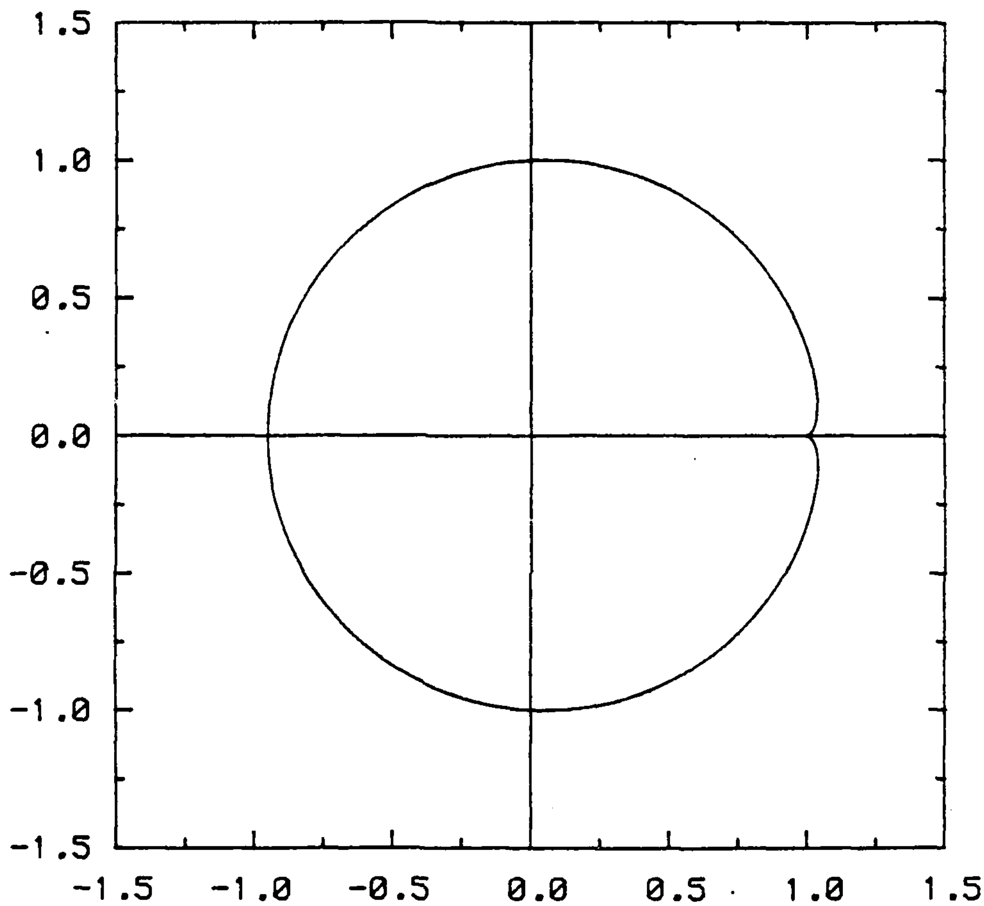

Fig 5.2(e): $G_{22}(e^{i\omega T})$; T = 0.05

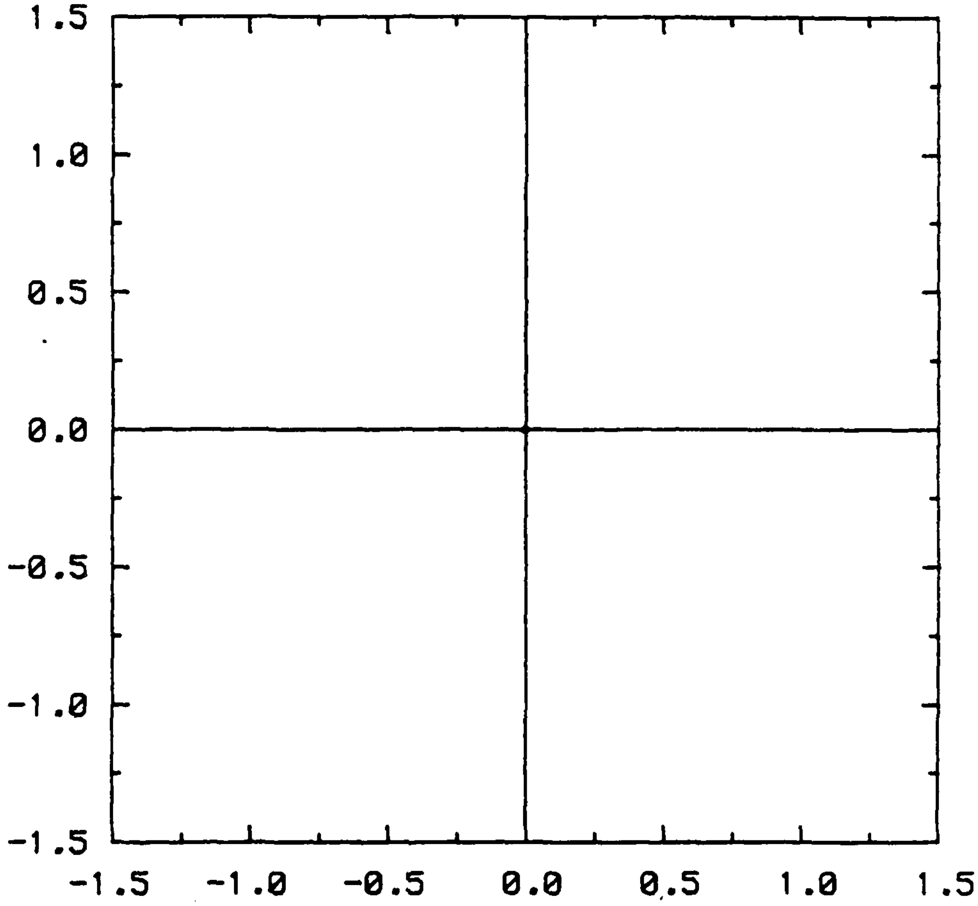

Fig 5.2(f): $G_{23}(e^{i\omega T})$; T = 0.05

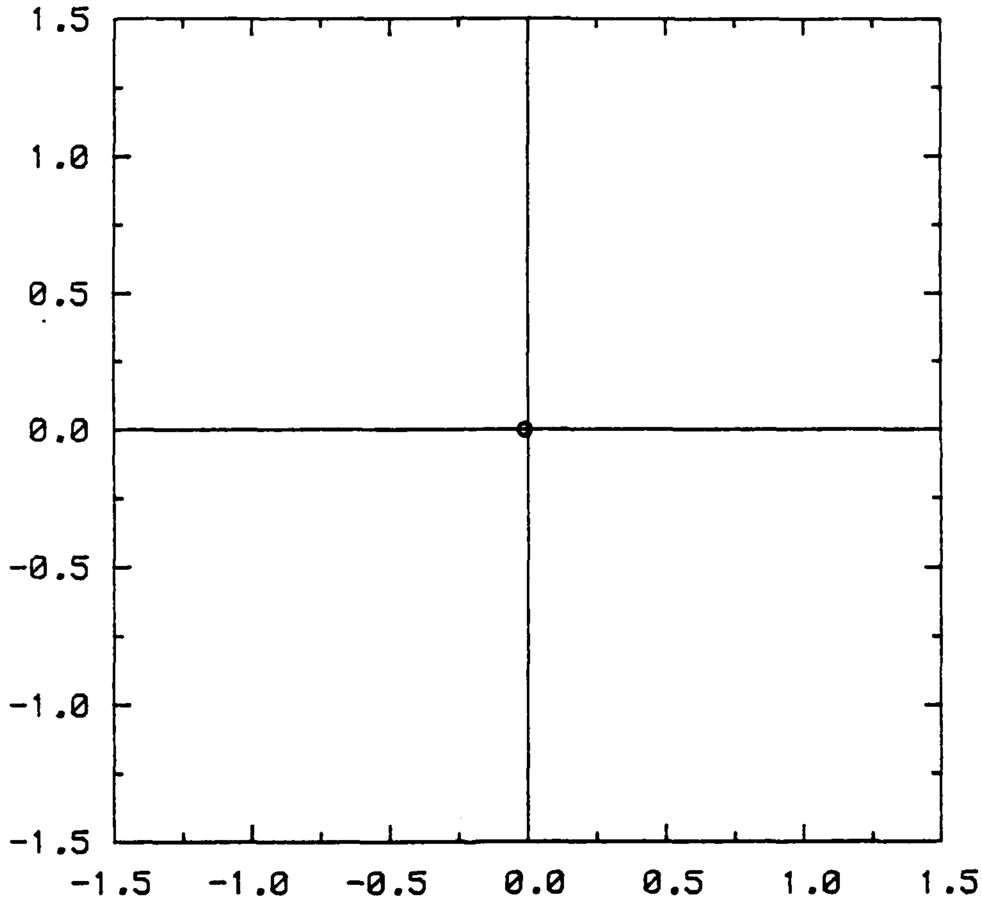

Fig 5.2(g): $G_{31}(e^{i\omega T})$; T = 0.05

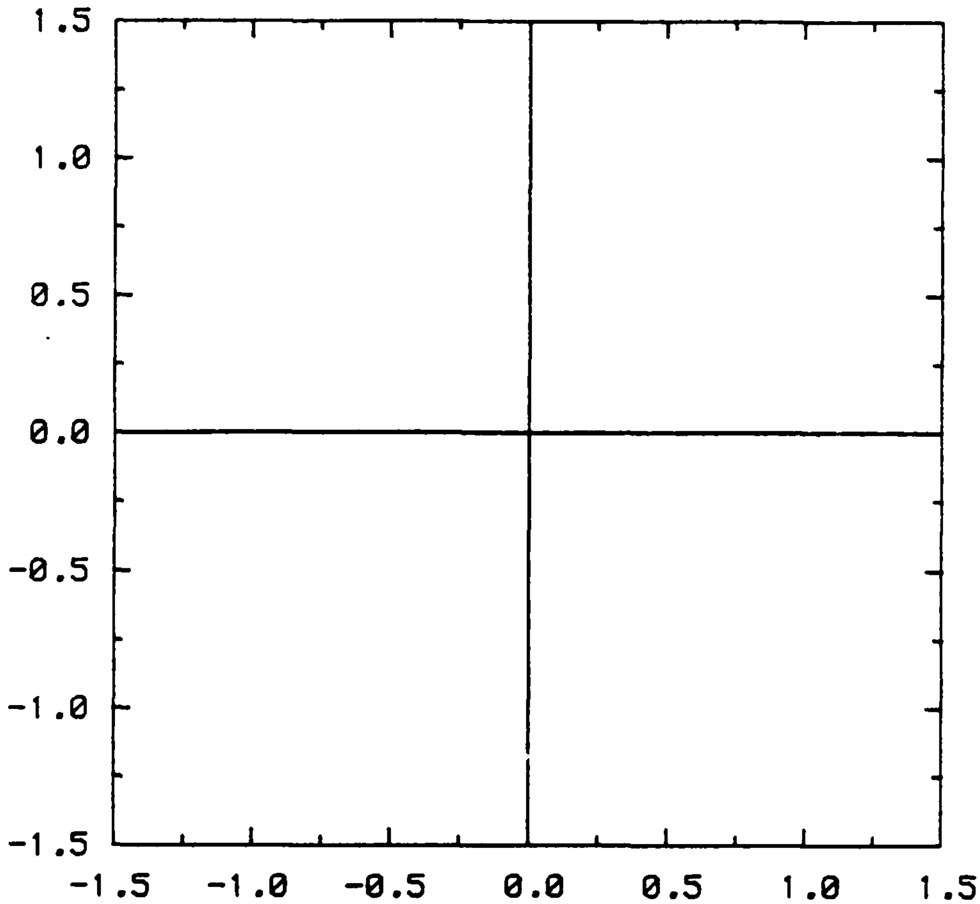

Fig 5.2(h): $G_{32}(e^{i\omega T})$; T = 0.05

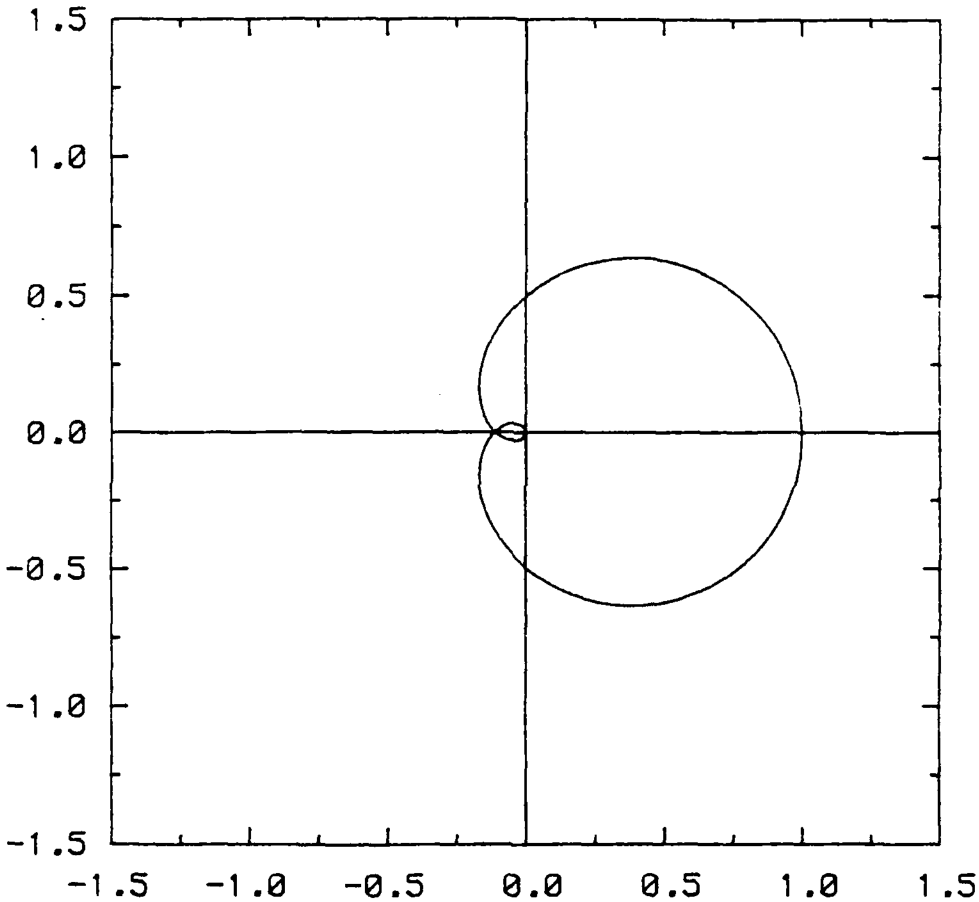

Fig 5.2(i): $G_{33}(e^{i\omega T})$; $T = 0.05$

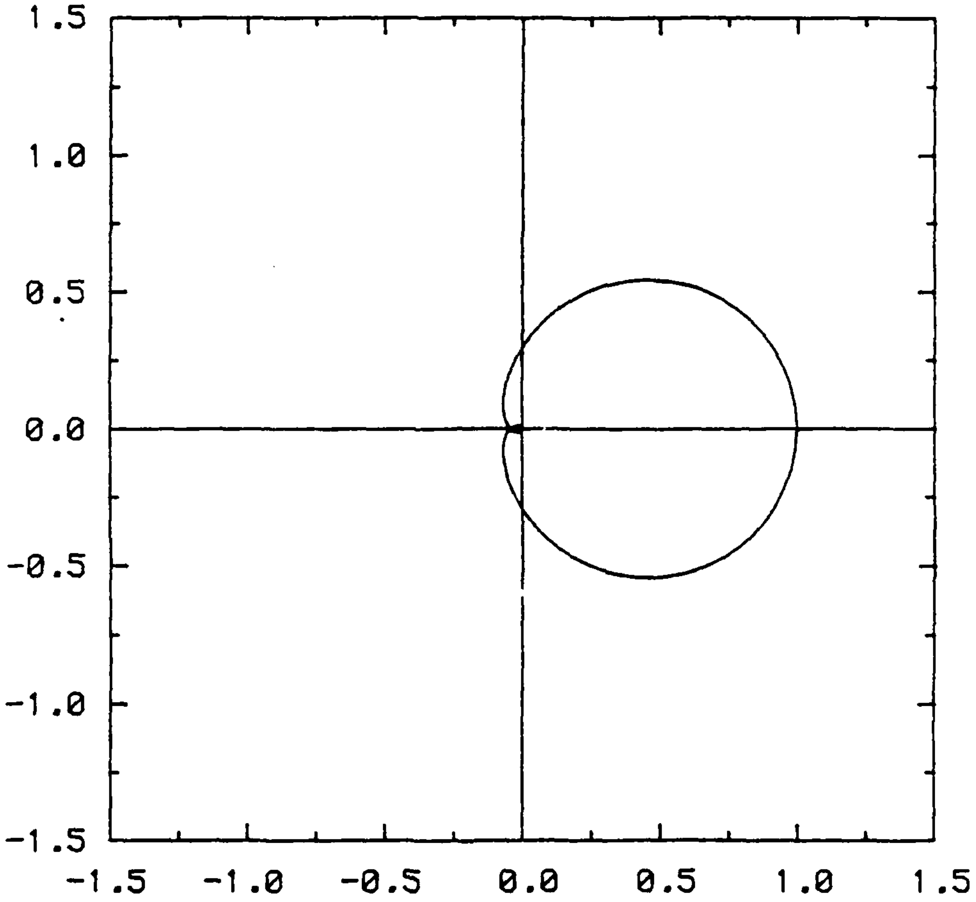

Fig 5.3(a): $G_{11}(e^{i\omega T})$; T = 0.02

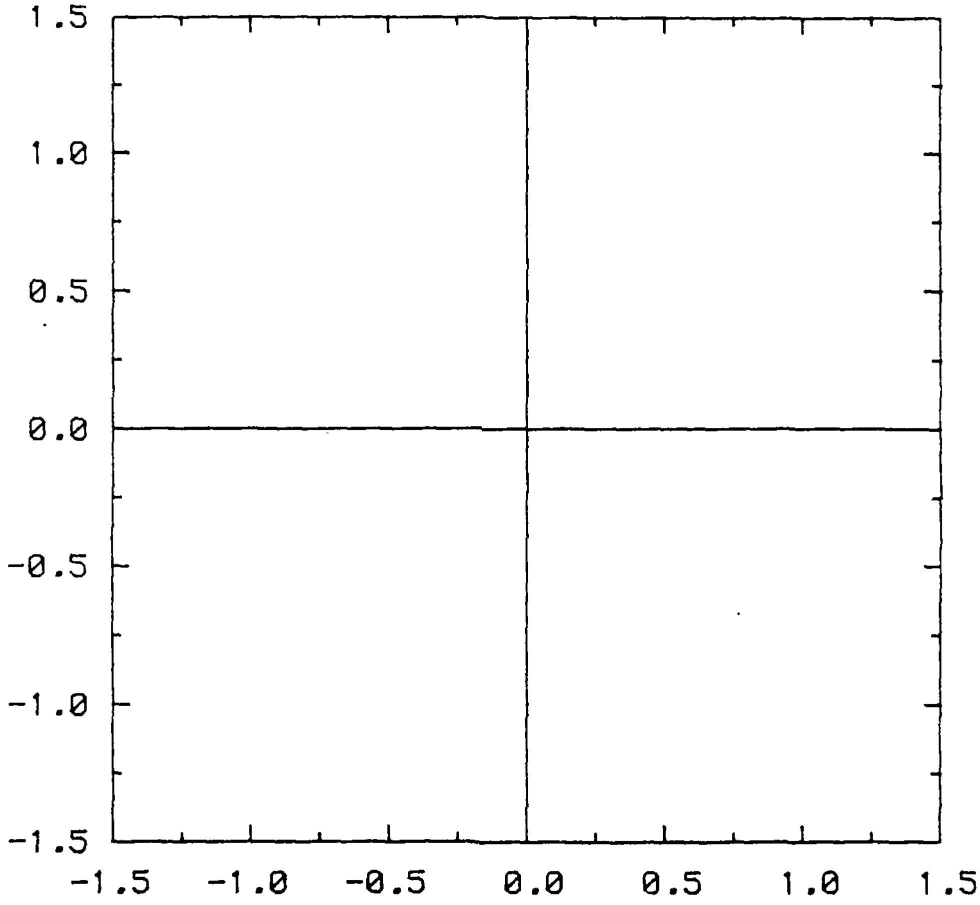

Fig 5.3(b): $G_{12}(e^{i\omega T})$; T = 0.02

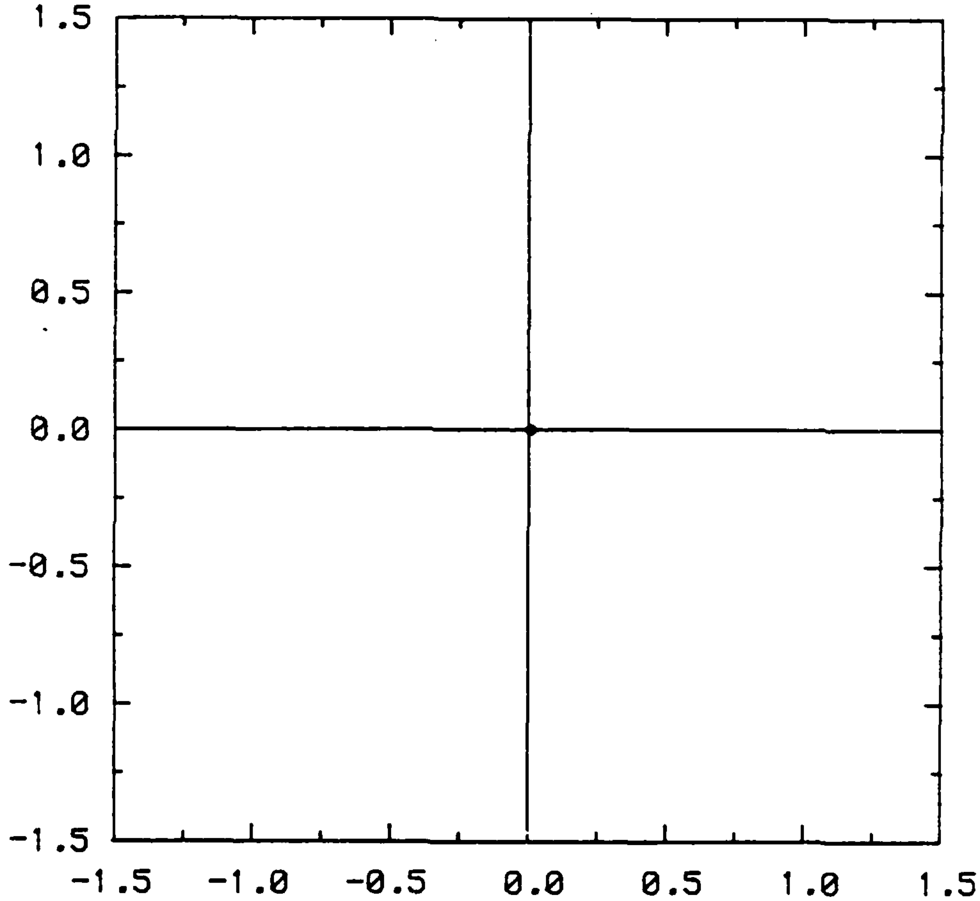

Fig 5.3(c): $G_{13}(e^{i\omega T})$; T = 0.02

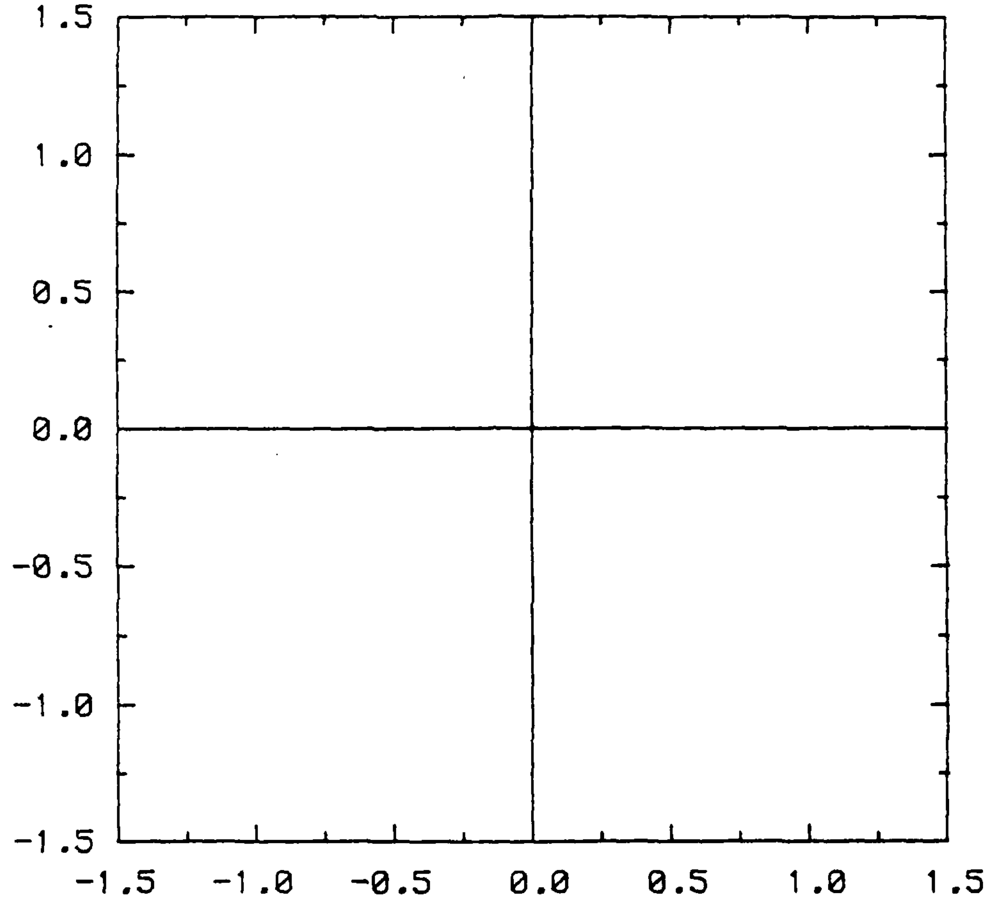

Fig 5.3(d): $G_{21}(e^{i\omega T})$; T = 0.02

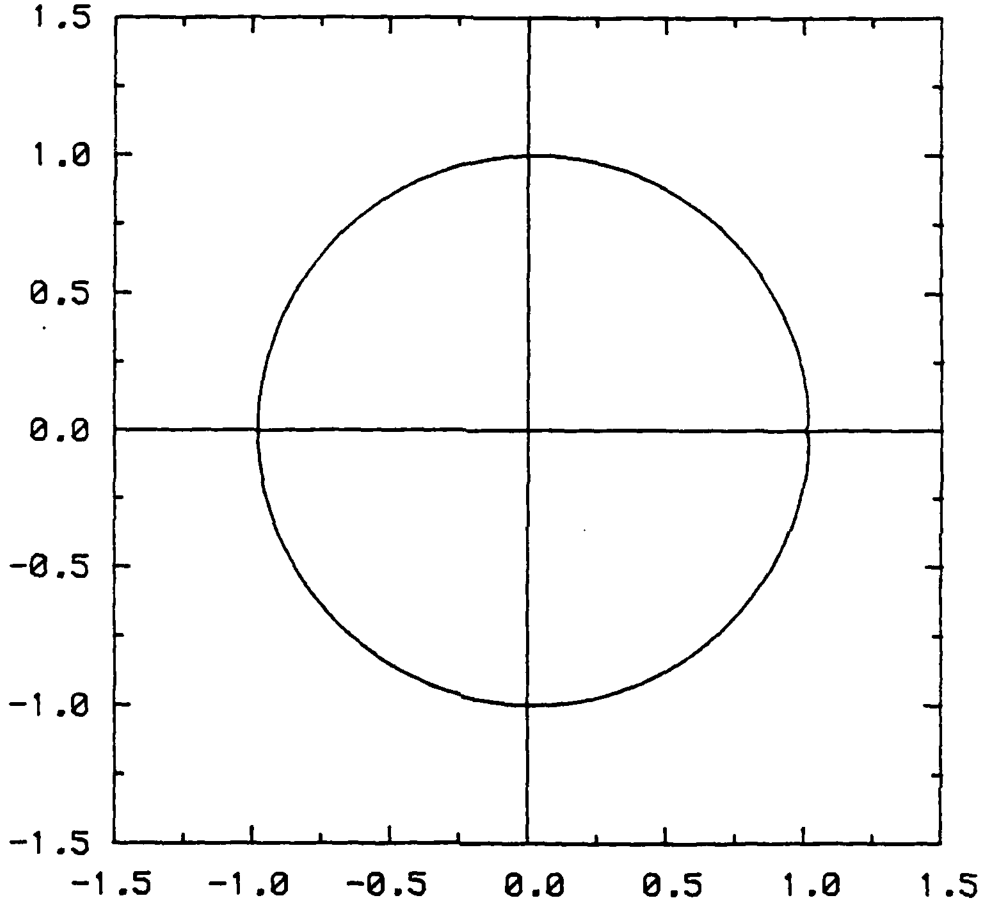

Fig 5.3(e): $G_{22}(e^{i\omega T})$; T = 0.02

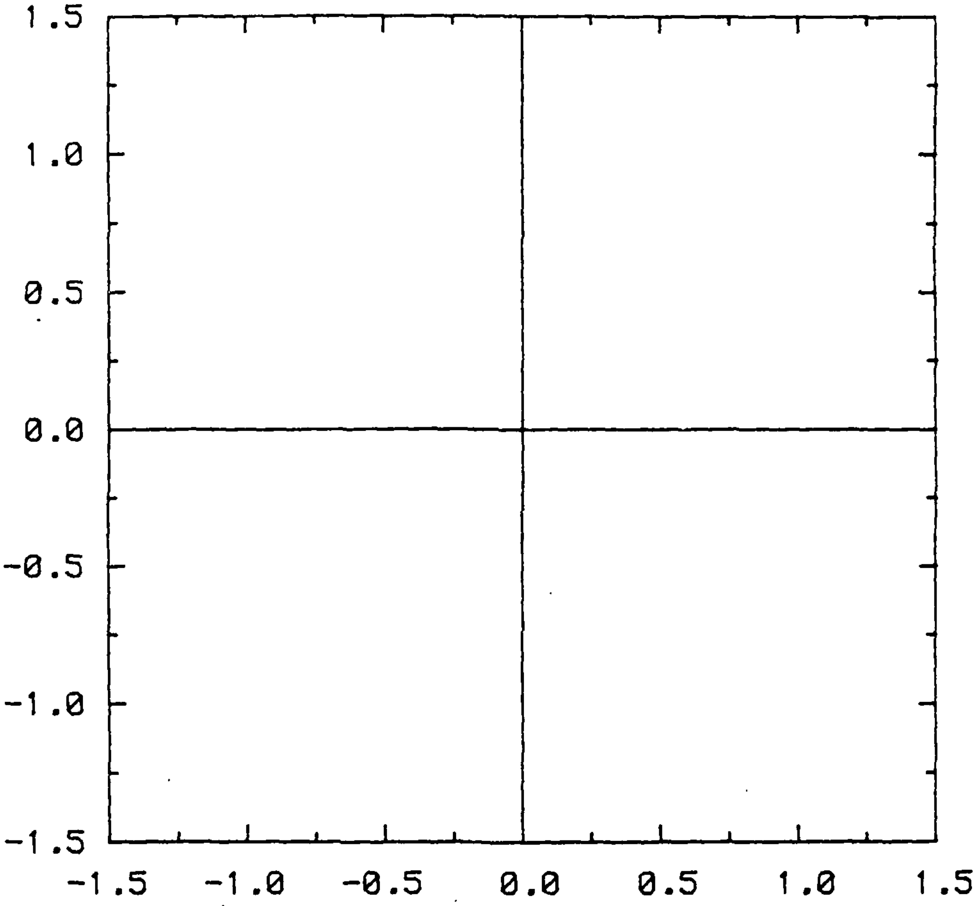

Fig 5.3(f): $G_{23}(e^{i\omega T})$; T = 0.02

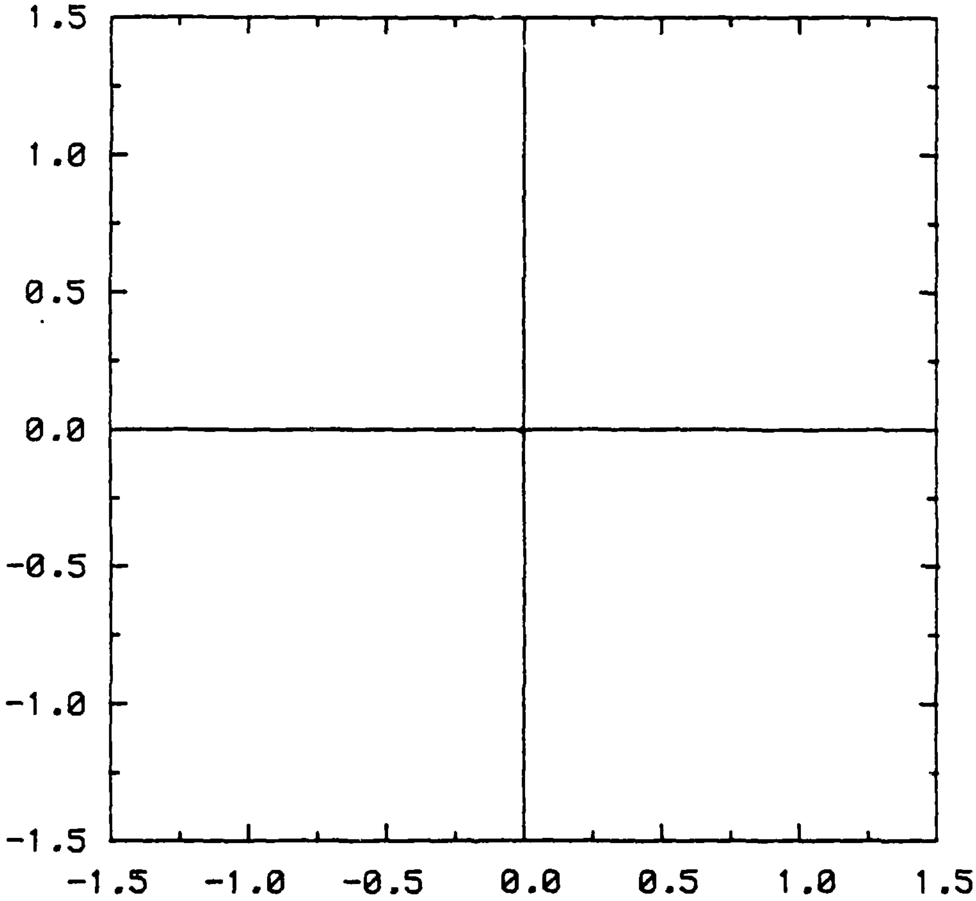

Fig 5.3(g): $G_{31}(e^{i\omega T})$; T = 0.02

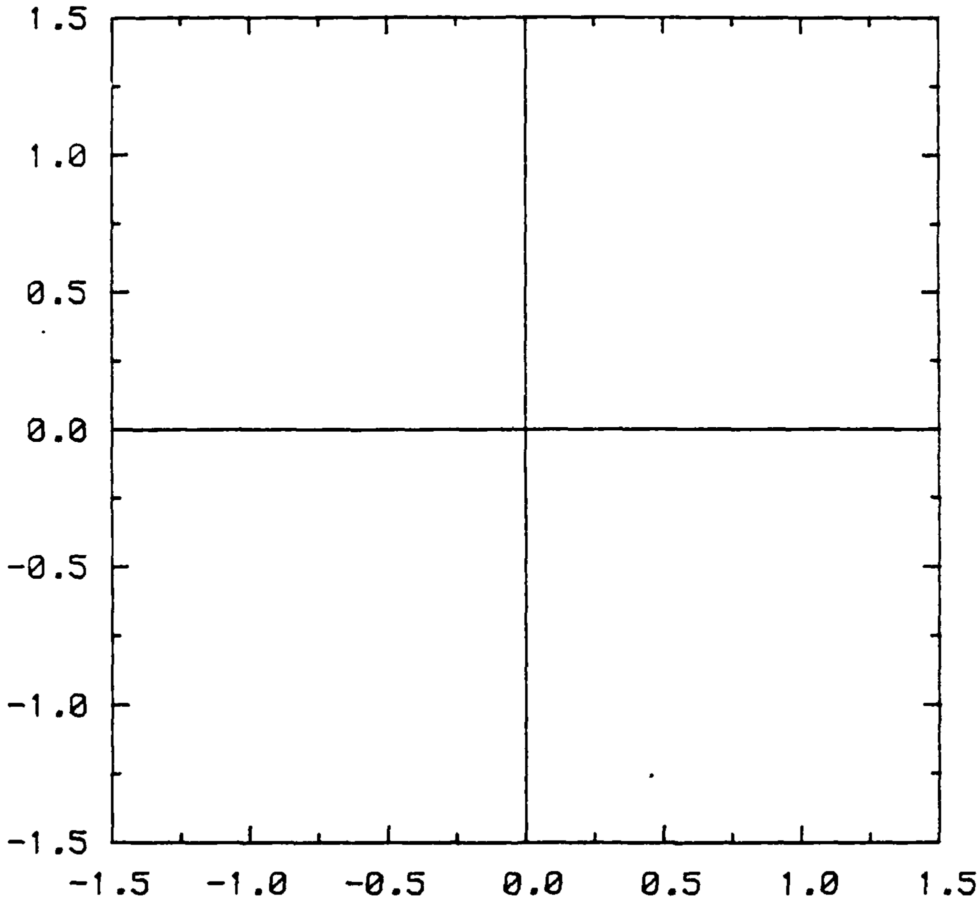

Fig 5.3(h): $G_{32}(e^{i\omega T})$; $T = 0.02$

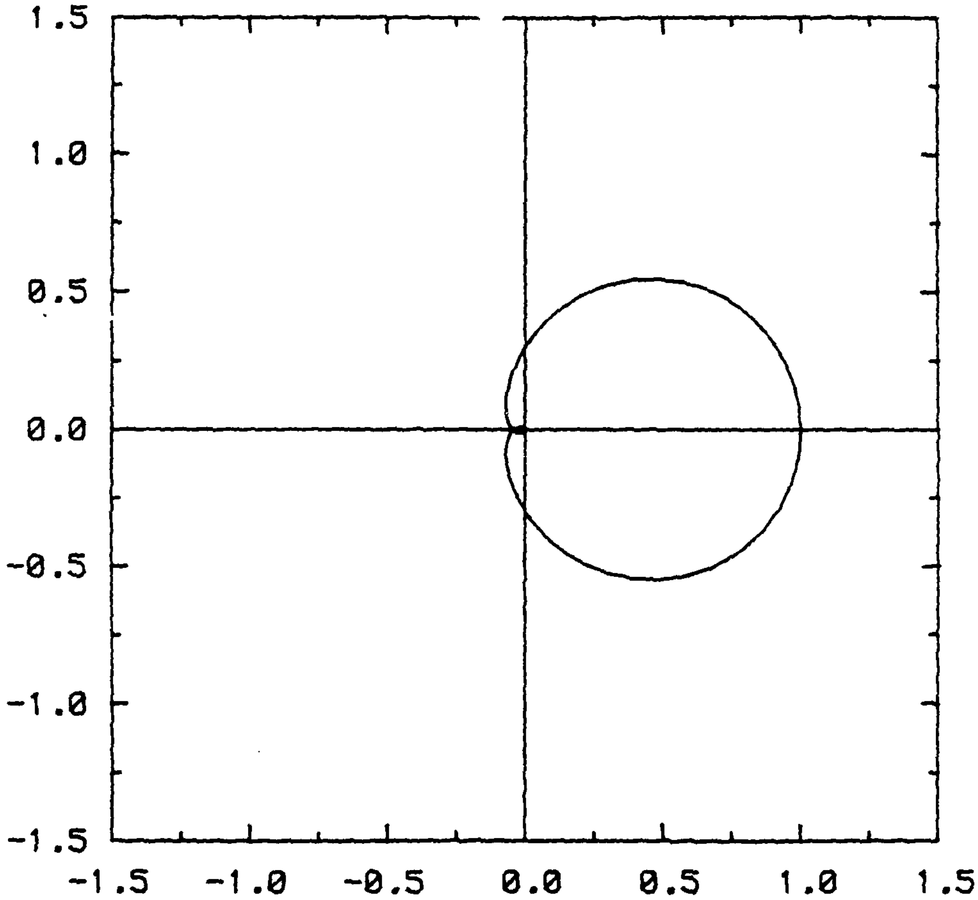

Fig 5.3(i): $G_{33}(e^{i\omega T})$; T = 0.02

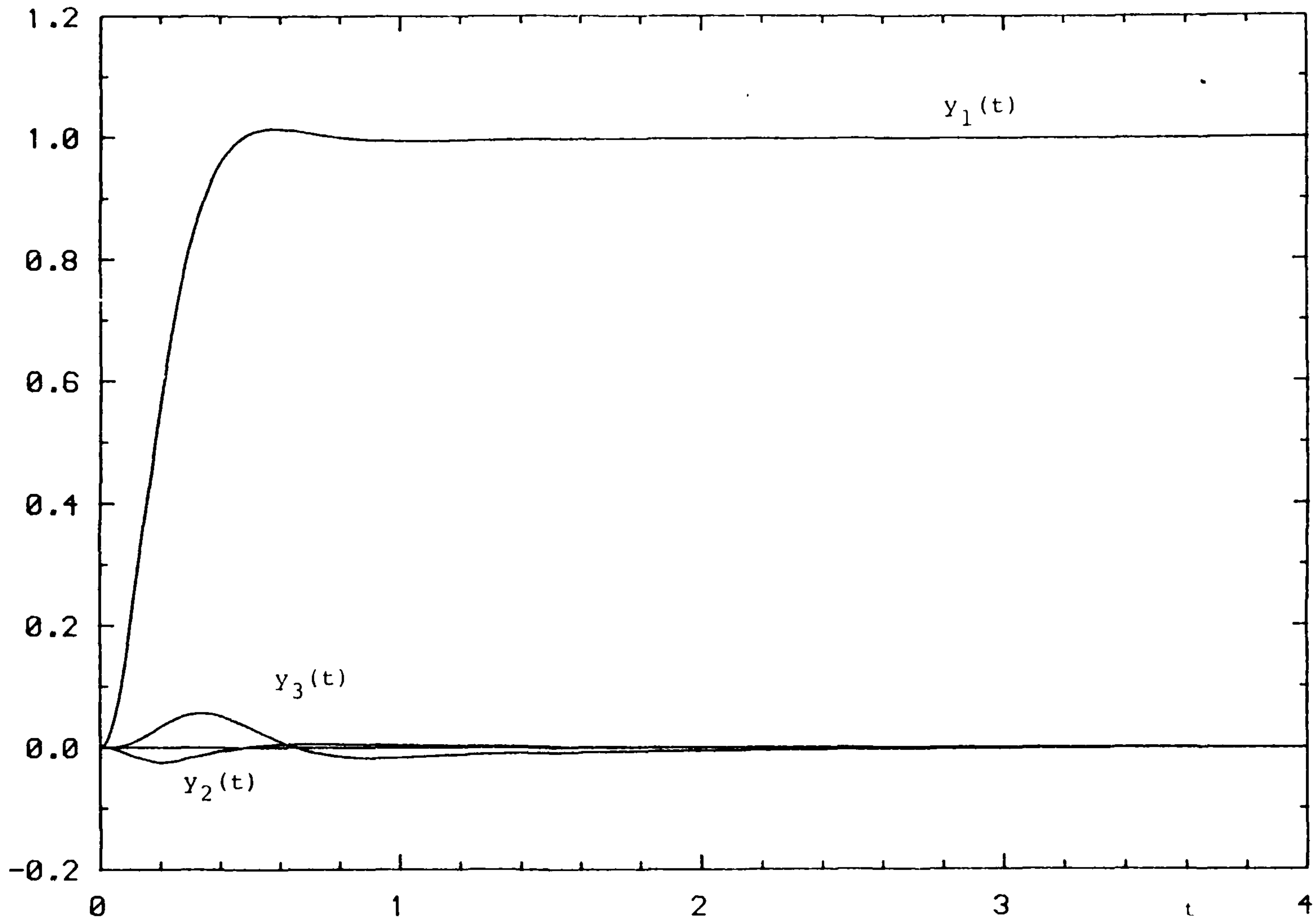

Fig 5.4(a): T = 0.10

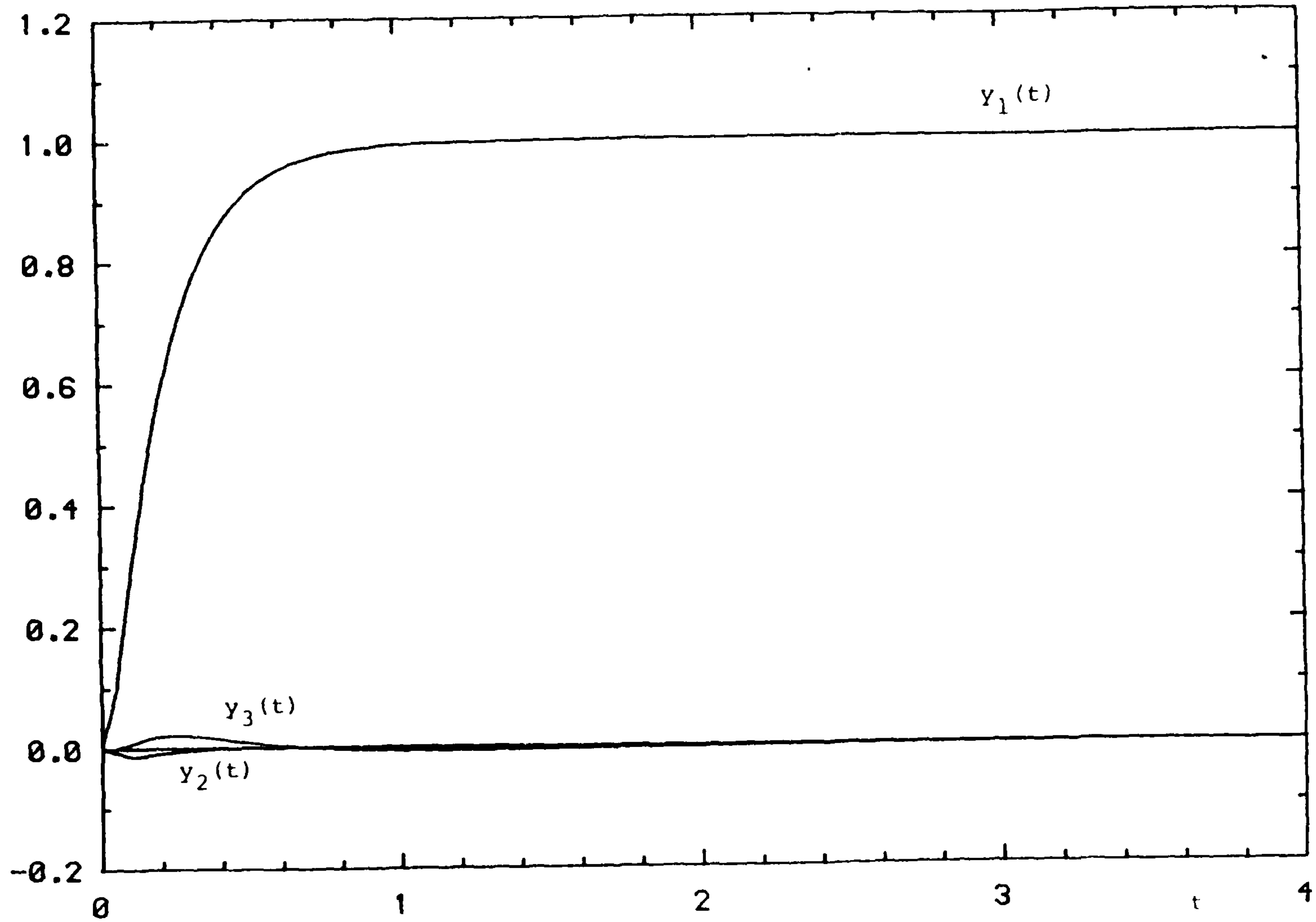

Fig 5.4(b): T = 0.05

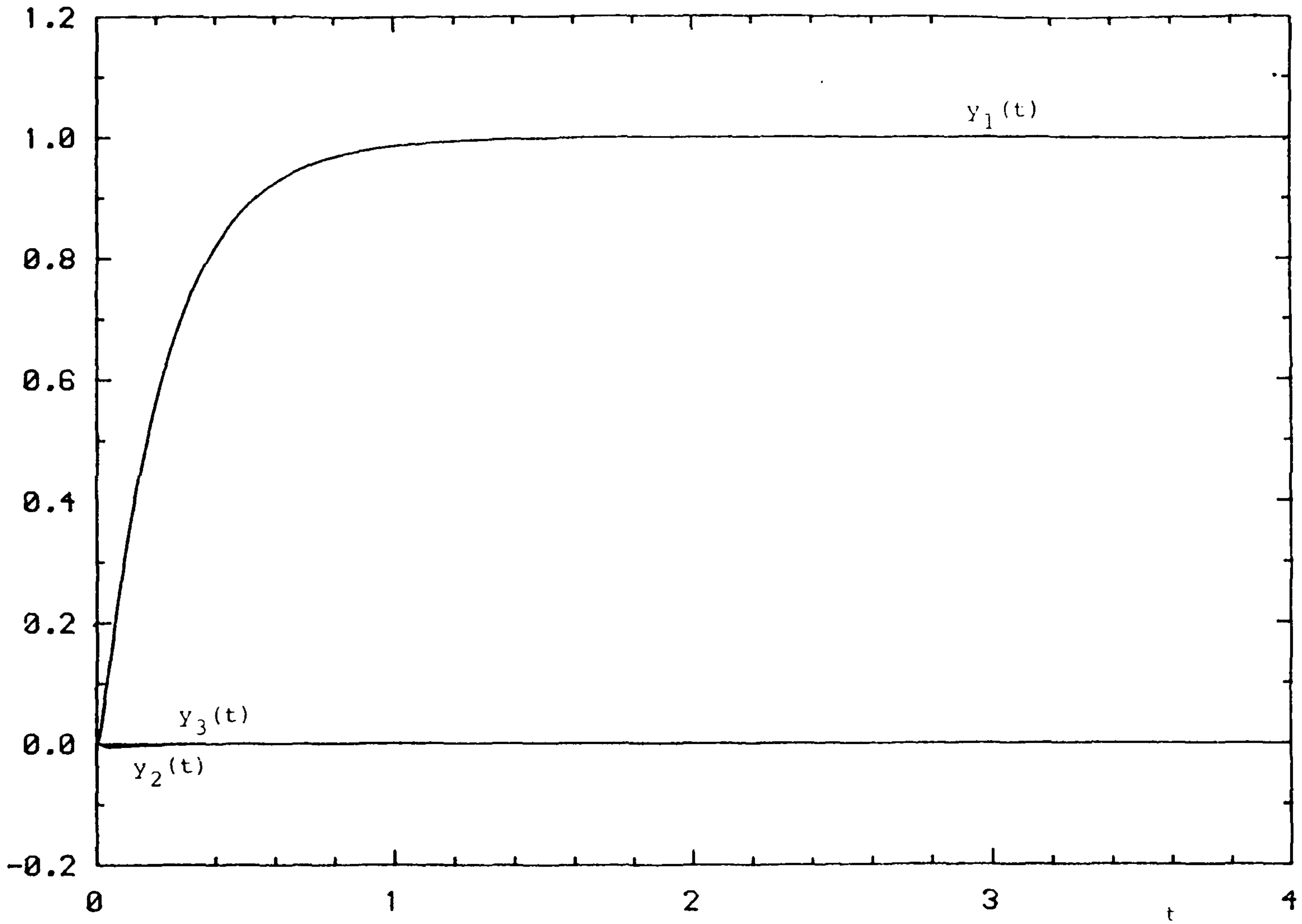

Fig 5.4(c): T = 0.02

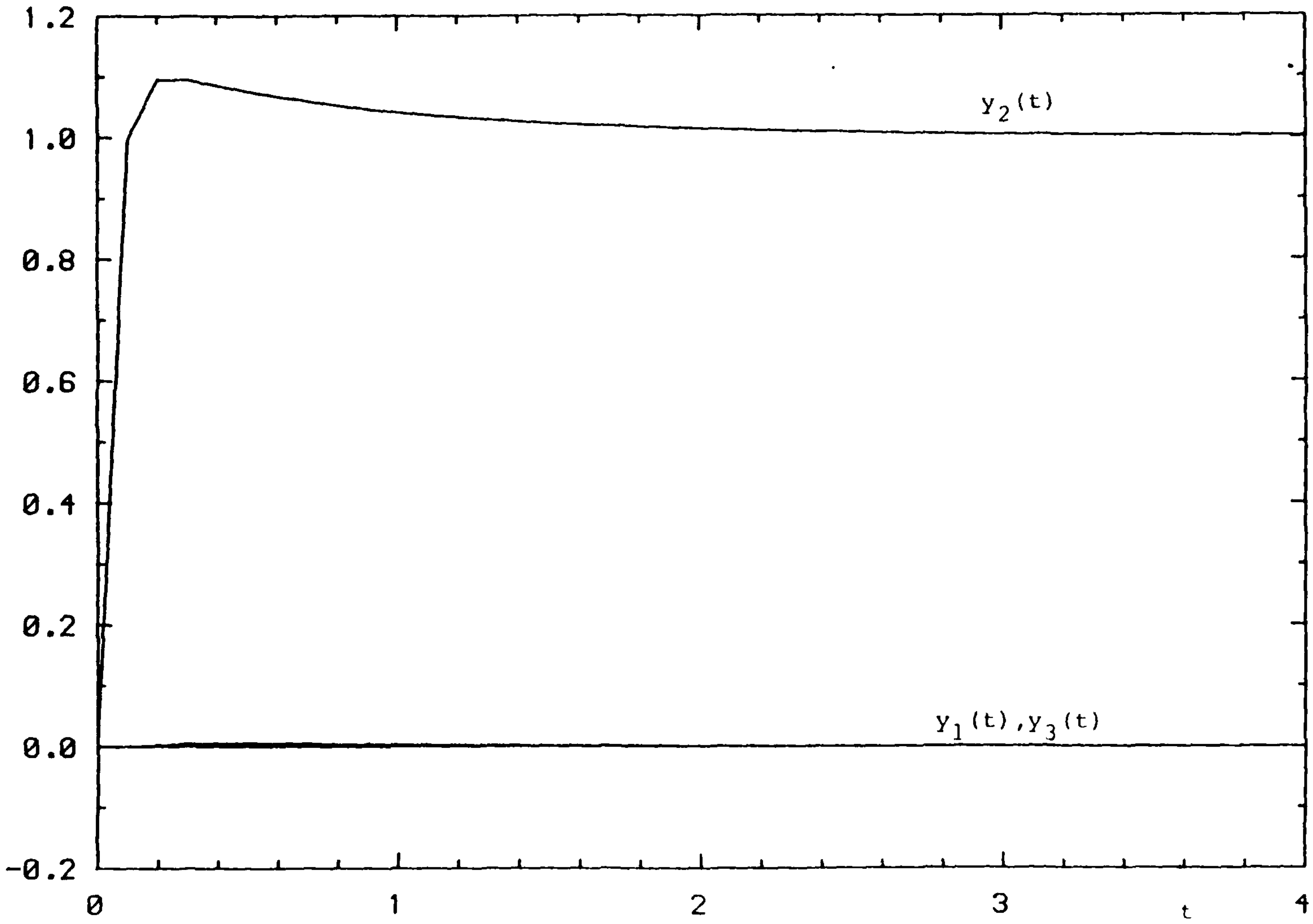

Fig 5.5(a): T = 0.1

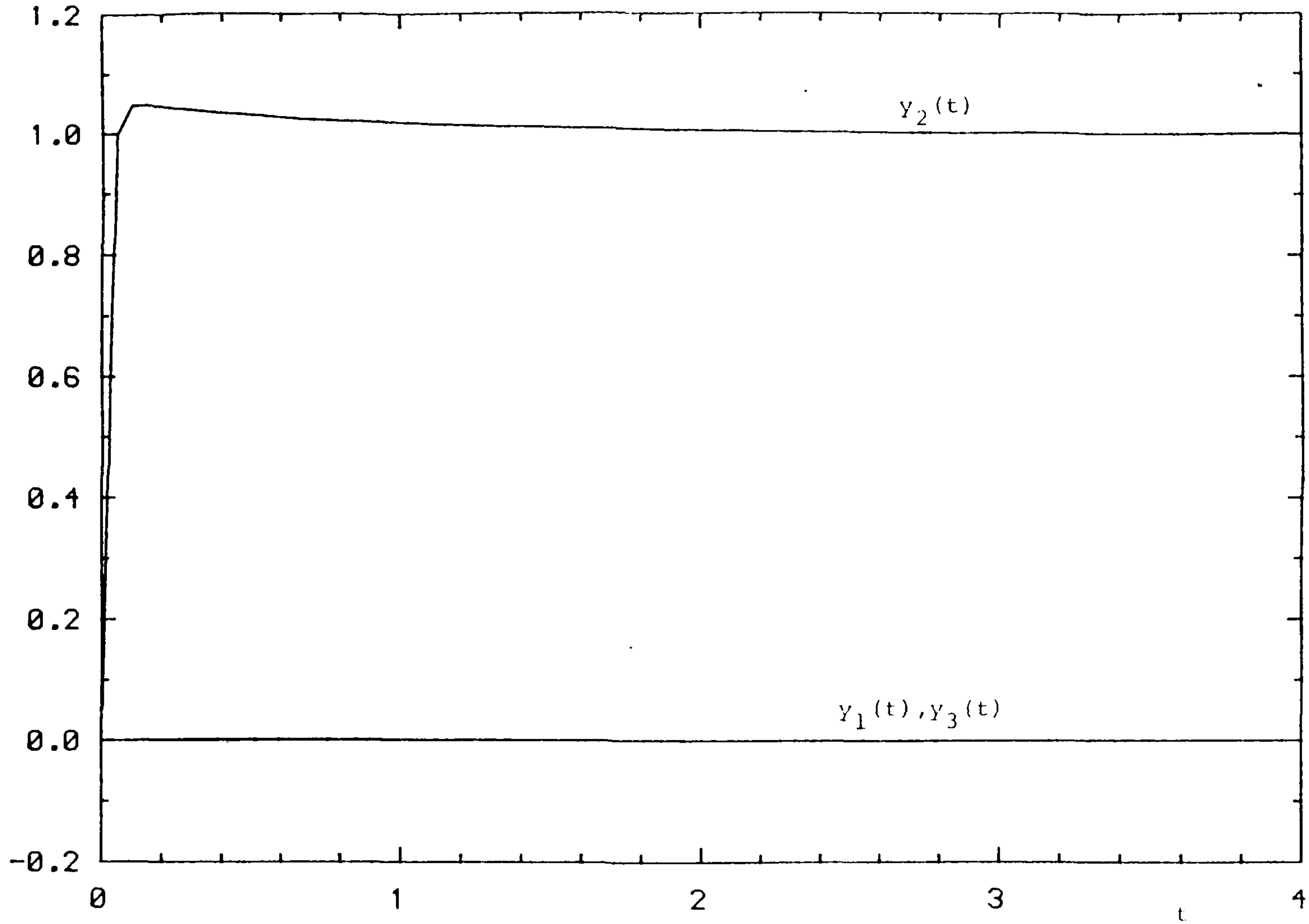

Fig 5.5(b): T = 0.05

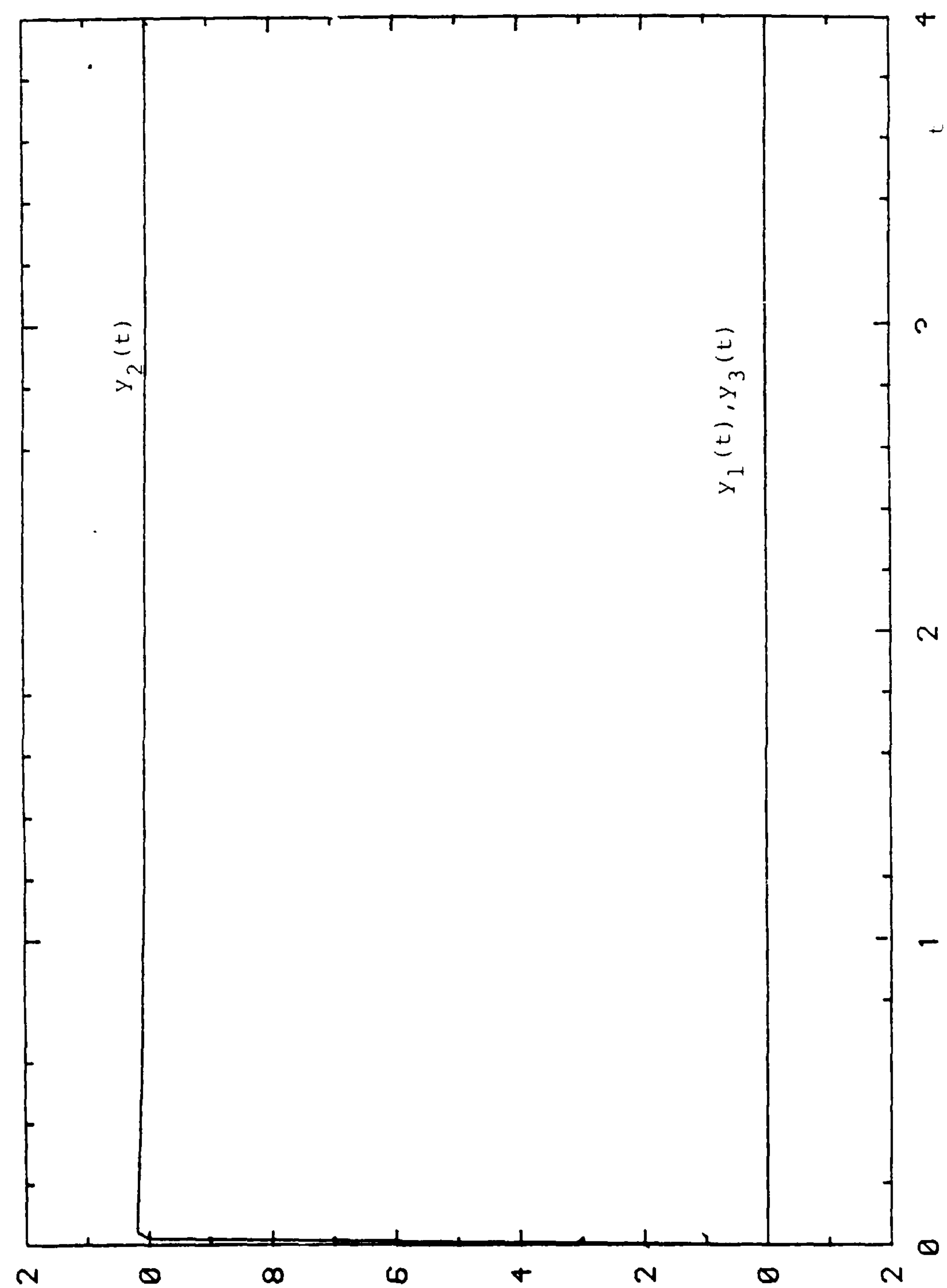

Fig 5.5(c): T = 0.02

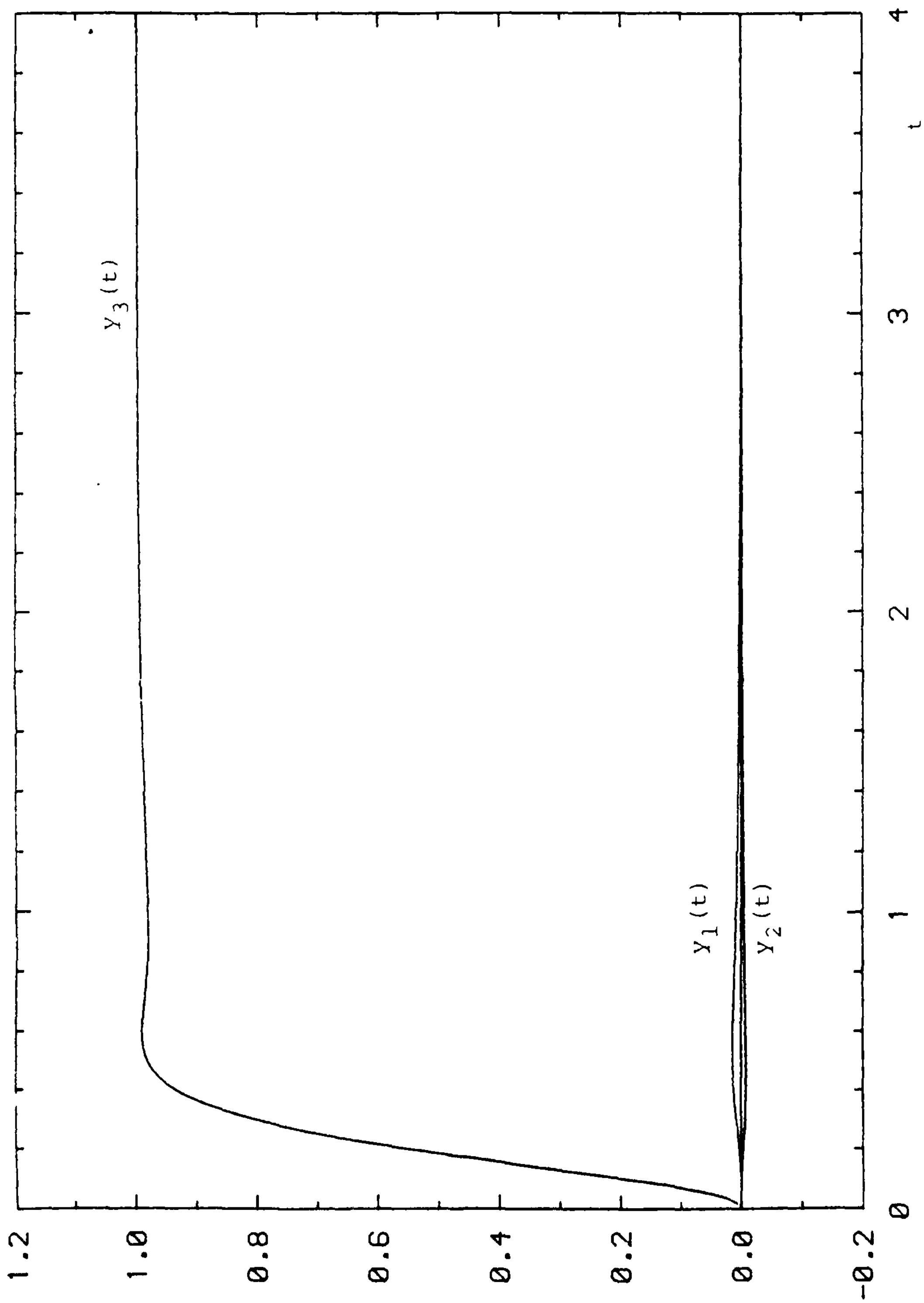

Fig 5.6(a): T = 0.1

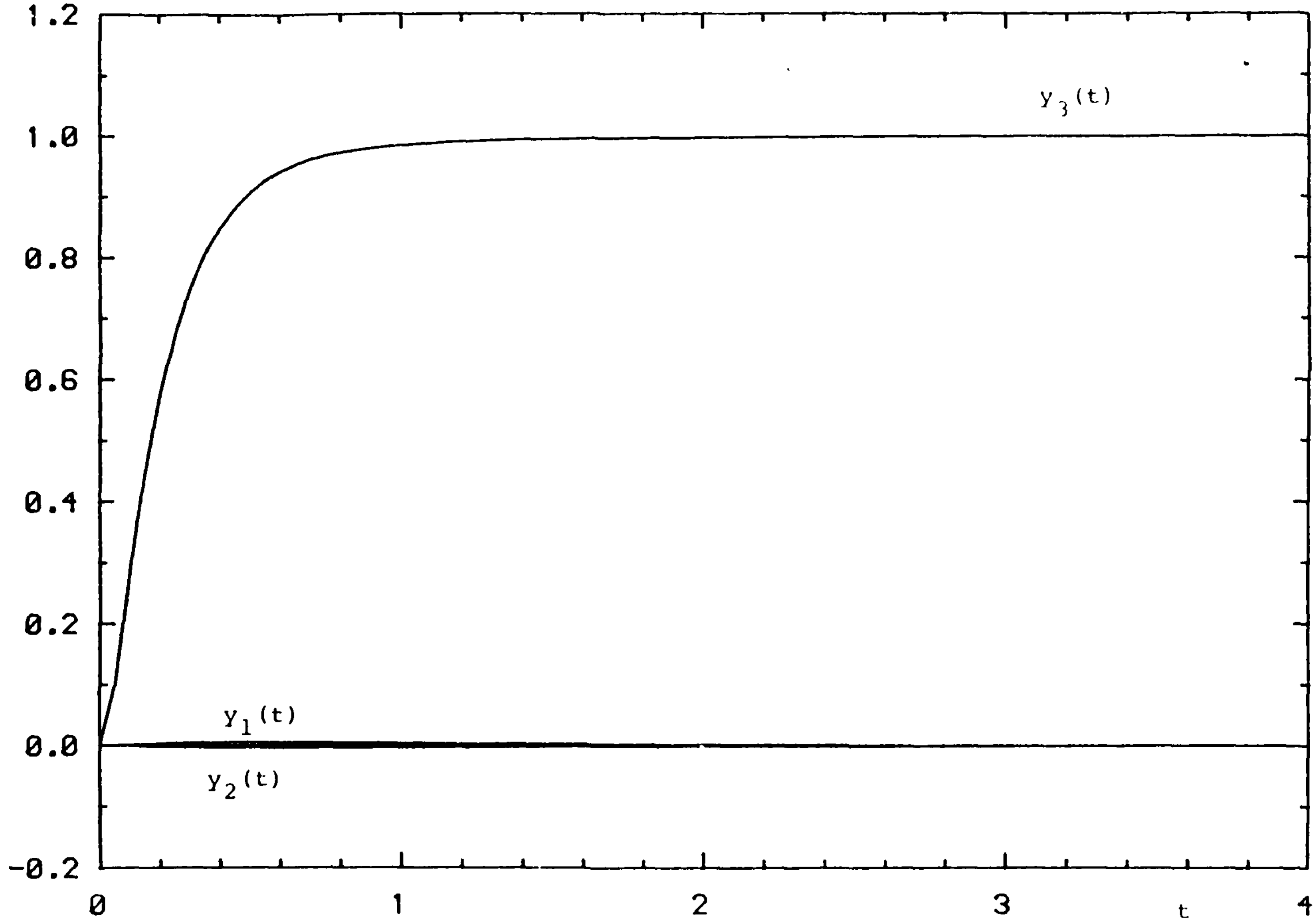

Fig 5.6(b): T = 0.05

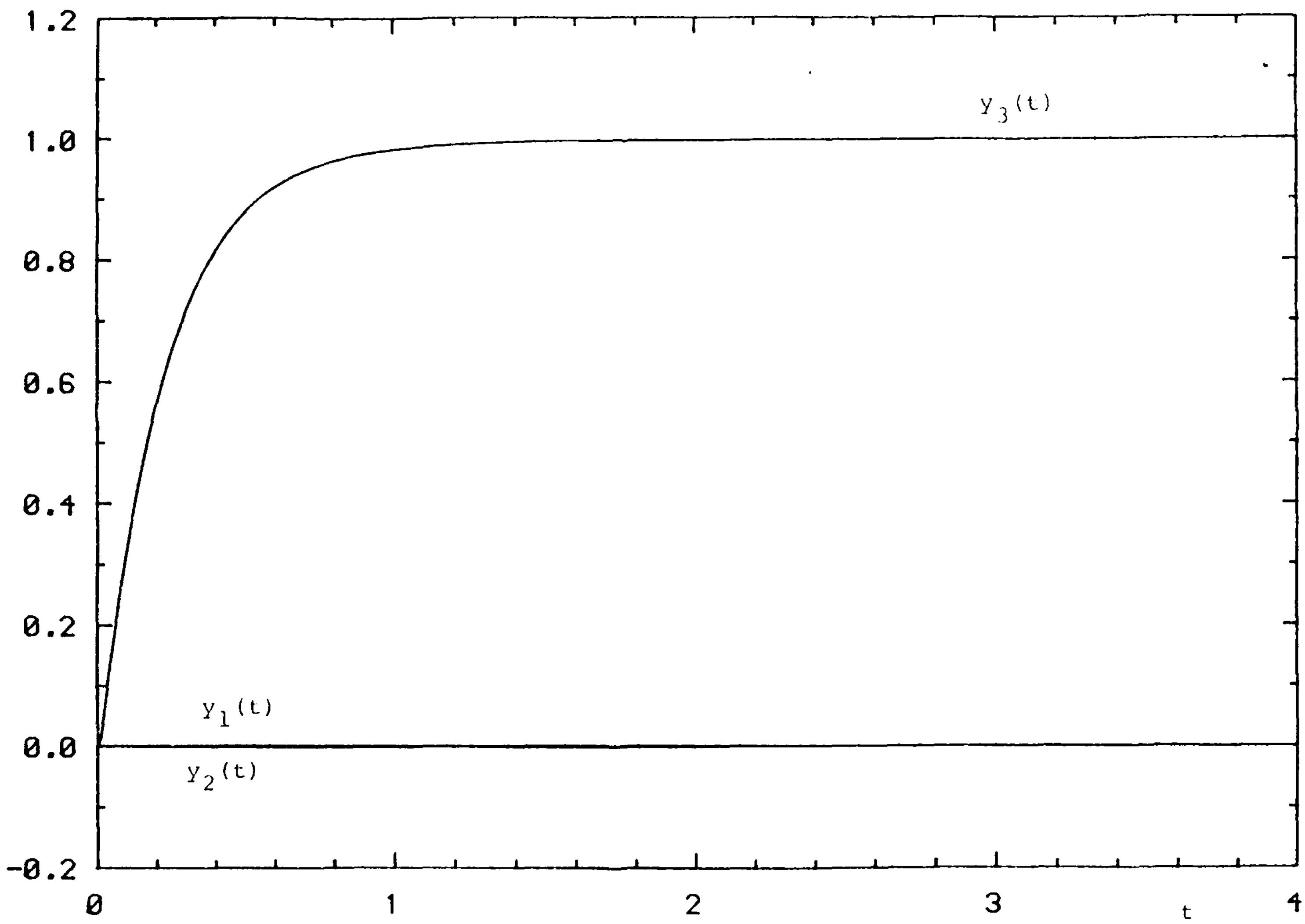

Fig 5.6(c): T = 0.02

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The conceptual and computational simplicity of singular perturbation methods (Porter and Shenton 1975) in the design of both analogue and digital tracking systems incorporating error-actuated controllers has been demonstrated. This demonstration has been effected by designing such controllers for plants which have previously been considered by other methods (MacFarlane and Kouvaritakis 1977, Kouvaritakis, Murray, and MacFarlane 1979) so that detailed comparisons between alternative design techniques can readily be made, at least in the case of error-actuated analogue controllers.

Moreover, further demonstration of the power of the design techniques described in this report has been provided by the design of direct digital flight-mode controllers for the F-16 aircraft (Porter and Bradshaw 1981). In addition, the fact that high-gain and fast-sampling error-actuated controllers remain highly effective in the presence of gross actuator nonlinearities has been conclusively demonstrated (Porter 1981a, Porter 1981b, Porter 1981c, Porter 1981d). Thus, the same basic design techniques have been shown to be applicable to both linear and nonlinear multivariable plants.

6.2 Recommendations

The controllers for the F-16 aircraft designed by singular perturbation methods (Porter and Bradshaw 1981) are extremely robust in the sense that the same controller remains effective throughout a wide range of different flight conditions. However, in case even higher performance characteristics are required, the use of adaptive controllers becomes unavoidable and the study of fast-sampling adaptive digital controllers is therefore recommended.

Finally, although fast-sampling digital controllers for the F-16 aircraft have been successfully implemented in the Engineering Dynamics and Control Laboratory at the University of Salford using microprocessor hardware (Garis 1981), the implementation of such controllers in actual aircraft and subsequent flight testing is recommended. Such practical implementation would obviously entail close liaison with the aerospace industry, based initially upon the wider dissemination of the design methodologies described in this report.

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